Comment on "Mass and Magnetic Dipole Shielding against Electrons of the Artificial Radiation Belt"

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The article by Bhattacharji and Michael shows that magnetic radiation shielding against electrons of the artificial belt is attractive when compared to solid shielding. It appears that they have chosen a relatively unfavorable configuration for their magnetic shield; as a result of this, they have emphasized a relatively unimportant aspect of the design, and substantially understated the attractiveness of their concept.

The shielded volume $V'$ around a dipole is proportional to $M^2$ where $M$ is the magnetic moment of the dipole. $M$, in turn, is proportional to $Ia^2$ where $I$ is the total circulating current and $a$ the radius of the coil. The mass of structural material $m_s$ required to contain the magnetic stresses is proportional to the magnetic energy stored by the device; this varies as $Ia^2$ for fixed geometrical ratios. For a given maximum current density of the superconducting material, the mass of superconductor $m_s$ required is proportional to $Ia$. Thus, for a given shielded volume (or magnetic moment), both masses can be indefinitely reduced by reducing $I$ and increasing $a$. Bhattacharji and Michael have worked, however, at a fixed magnetic field strength of 50 kgauss. That is, they have fixed the ratio $I/a$. From the foregoing, it can be seen that this implies that $m_s = 1/2a$, and this proportionality appears in their Eq. (6) and also in their Fig. 2. Much lower weights than they quote can be achieved by reducing the level of both the current and magnetic field, and by enlarging the geometry. The optimization actually performed by Bhattacharji and Michael deals only with the aspect ratio of the solenoid providing the magnetic moment. Based on considerations of structural efficiency, they arrive at the conclusion that the optimum aspect ratio is roughly square. We shall see below that structural considerations are, in any event, unimportant for the shielding purposes under consideration.

One cannot, of course, achieve indefinitely lower values for $m_s$ and $m_s$ by reducing $I$ and increasing $a$. The shielded volume declines sharply if $a$ is larger than the Stormer radius $a_s$ because the magnetic field in the shielded volume becomes significantly different from the field due to a dipole. Minimum weight for a given shielded volume occurs when these lengths are roughly equal, thus

$$a_s = \left(\frac{4\pi \rho \delta^2 \epsilon M}{12}\right)^{1/3} = \left(4\rho \mu_0 I \delta^2 \right)^{1/3}$$

in mks units. For 10 Mev electrons, the momentum to charge ratio $p/e$ is 0.035. Therefore, a shield of any size designed to protect against 10 Mev electrons should operate with fixed total current of about 10$^3$ amp.
Since \( m_{m_2} = \rho m \) and \( m_{m_1} = \rho n \), then, whatever the size, there is a critical current below which \( m_{m_1} < m_{m_2} \). Using the structural material chosen by Bhattacharjie and Michael and a critical current density of \( 10^9 \) amp/cm, the current is roughly \( 10^7 \) amp. Since this is 100 times greater than the current required for shielding, we expect the structural weight to be about \( 1\% \) of the superconducting material weight, independent of the size. The whole question of structure is, therefore, unimportant in the context of 10 MeV electrons.

Using the value \( \rho = 0.6 \) taken from Ref. 2 and using the same reference to estimate the true shielded volume for a roll of this type, one arrives at the conclusion that the weight of the shield lies almost entirely in the superconducting material; this weight is roughly given by

\[
m_{m_2} = 2.85^{1/2}
\]

The structural weight is a negligible fraction of the superconducting weight. Note the correct power dependence of \( m_0 \) on \( V \), when the optimization is done at constant current. These weights are lower than those quoted by Bhattacharjie and Michael by a factor of 2 when \( V = 100 \) and by a factor of 50 when \( V = 1000 \). With weights as low as these, it is likely that considerations of the area of cryogenic surface required would exert a significant influence on the design.

References