Sensitivity Study and Optimization of a 3D Electric Impedance Tomography Prostate Probe

A. Borsic¹, R. Halter¹, Y. Wan¹, A. Hartov¹, K. D. Paulsen¹
Thayer School Of Engineering, Dartmouth College, 8000 Cummings Hall Hanover, NH 03755, US
E-mail: Andrea.Borsic@Dartmouth.edu

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Abstract. In the current clinical practice the primary diagnostic method for testing for prostate cancer is ultrasound-guided biopsy. In this paper we consider using a sonolucent array of electrodes, printed on a thin Kapton layer and positioned on the imaging window of a transrectal ultrasound probe, as a method for providing coregistered electrical an ultrasound imaging of the prostate. As the electrical properties of malignant tissues have been shown to differ significantly to from benign tissues, the estimation of the electrical properties is expected to be helpful in distinguishing certain begin pathologies from cancer and in improving the detection rate that current biopsy methods provide. One of the main difficulties in estimating electrical properties of tissues with this electrode configuration is the rapid decay of the sensitivity with distance from the sensing array. In order to partially overcome this difficulty we propose to use prior information from US. Specifically we intend to delineate the boundaries of the prostate from US, to subdivide the organ in a small number of voxels and to estimate the conductivity as constant on each of these subvolumes. We use a 3D forward model based on the finite element method for studying the sensitivity of a simulated segmented prostate for three different electrode array designs. The three designs present different electrode areas and inter–electrode gaps. Larger electrodes are desirable as they present a better contact, but we show that as they result in smaller inter–electrode gaps, shunting currents can be significant and the sensitivity is reduced. Because our clinical measurement system employs a single current source, we consider tetrapolar measurement patterns for evaluating these electrode configurations. The three designs present different electrode areas and inter–electrode gaps. Larger electrodes are desirable as they present a better contact, but we show that as they result in smaller inter–electrode gaps, shunting currents can be significant and the sensitivity is reduced. Because our clinical measurement system employs a single current source, we consider tetrapolar measurement patterns for evaluating these electrode configurations. Optimal measurement patterns are well defined for adaptive systems, where multiple currents are injected at the same time. For the electrode array designs we consider, which are three dimensional, there are no established systematic methods for forming sets of linearly independent tetrapolar measurement patterns. We develop a novel method for automatically computing a full set of independent tetrapolar measurement patterns that maximizes the sensitivity in an ROI. We use these patterns in the forward modeling and sensitivity studies. In addition to the electrode arrays on the probe, we study the use of a further configuration, where a distal electrode is positioned on the exterior of the body and used for current injection.

Keywords: Electrical Impedance Tomography, Ultrasound, Finite Element Method, Electrode Modeling, Sensitivity Optimization, Three Dimensional, Prostate Imaging
1. Background

Approximately 28,600 American men are expected to die from prostate cancer in 2008, with an additional 186,320 diagnosed with the disease (Jemal, Siegal, Ward, Yongping, Xu, Murray & J. 2008). The primary tool used for prostate cancer screening is measurement of prostate-specific antigen (PSA) in blood serum. These blood tests unfortunately lack sufficient specificity to definitively diagnose the disease and instead act as surrogate markers for clinical testing (de Koning, Auvinen, Sanchez et al. 2002). Definitive diagnosis is made through ultrasound-guided biopsy of the prostate. Unfortunately, only one in four men biopsied are found to have prostate cancer (Catalona 2004) because 1) they have an elevated PSA resulting from some other physiological mechanism (e.g. benign prostatic hyperplasia (BPH), acute prostatitis, acute urinary retention, previous biopsy and transurethral resection of the prostate, and/or ejaculation) (Price, Allard, Davies, Dawnay, Duffy, France, Mandarino, Ward, Patel, Sibley et al. 2001) or 2) they have cancer, but it was not sampled during the biopsy procedure.

Biopsy procedures are typically performed using transrectal ultrasound (TRUS) to guide needle placement. Typically, 6-12 cores are extracted; if additional hypo- or hyperechoic regions are visible, these too are often sampled; however, there is no clinical evidence that this extra sampling improves the detection rate of the disease. In fact, it is well-established that benign lesions within the prostate including BPH or dense stromal nodules display hypo- and/or hyperechoic signatures that are difficult to distinguish from cancer (Coley, Barry, Fleming & Mulley 1997). Currently, the available prostate imaging modalities including TRUS, endorectal MRI, and pelvic CT, are used only for staging of the disease (i.e. assessing extracapsular extension, rectal wall or seminal vesicle involvement, etc) and not as screening tools or for lesion specific biopsy guidance.

A number of investigations have suggested that the electrical properties of prostate cancer are significantly different from those of benign tissues within the gland (Lee, Roberts, Smith, Ko, Epstein, Lecksell & Partin 1999), (Halter, Hartov, Heaney, Paulsen & Schned 2007), (Halter, Hartov, Paulsen, Schned & Heaney 2008), (Halter, Schned, Heaney, Hartov, Shunz & Paulsen 2008). Specifically, the conductivity of malignant tissue in the prostate has been observed to be less than that of benign tissues. We propose to image these electrical properties using transrectal electrical impedance tomography (TREIT) coupled with TRUS to provide a more specific modality that is potentially able to act as a screening device secondary to PSA-monitoring or serve as an imaging technique with enhanced lesion specificity for biopsy guidance. This probe consists of a sonolucent electrode array positioned over the acoustic window of a 3D TRUS device which provides coregistered electrical property and ultrasound imaging.

We use a tetrapolar EIT configuration with 32 channels in which a current is driven between two electrodes and single-ended voltages are sampled on the remaining electrodes. The source current and sensed voltages are used as the boundary values for a numerical algorithm that estimates the internal electrical properties. Critical to the
design of this multimodal imaging probe is definition of the optimal measurements and of the electrode array design that maximizes measurement sensitivity within the prostate. Here, we present the theoretical framework for establishing these system parameters. Specifically, we describe the forward model and define sensitivity, discuss the device and the FEM (Finite Element Method) framework employed, develop an algorithm for forming the optimal tetrapolar measurement patterns, and evaluate probe sensitivity for a number of different electrode configurations.

2. Forward Model and Sensitivity

In electric impedance imaging, the forward problem is modeled with a low-frequency approximation, where the electric field is considered conservative and the conduction currents are dominant with respect to the displacement currents. This model is described by the classic partial differential equation:

$$\nabla \cdot \sigma \nabla u = 0 \quad \text{on } \Omega$$

(1)

where $\sigma$ is the conductivity or admittance of the body to be imaged, $u$ is the electric potential, and $\Omega$ the imaging domain. The Complete Electrode Model (Somersalo, Cheney & Isaacson 1992) defines the boundary conditions at the electrodes. For a set of $L$ electrodes the vector $\{I_1 \ldots I_L\}$ describes the applied current intensities, and the vector $\{V_1 \ldots V_L\}$ the resulting voltages. At each electrode $\ell$ the flux of the current density must equal the injected current

$$I_\ell = \int_{\partial \Omega_\ell} \sigma \frac{\partial u}{\partial n} \quad \ell = 1 \ldots L$$

(2)

where $\partial \Omega_\ell$ is the portion of the boundary underneath the $\ell$–th electrode and $n$ is the inward normal. Additionally, for each electrode the sum of the potential on the body and the potential drop across the contact impedance, must equal the potential at the electrode:

$$u + z_c \sigma \frac{\partial u}{\partial n} = V_\ell \quad \text{on } \partial \Omega_\ell \quad \ell = 1 \ldots L$$

(3)

where $z_c$ is the contact impedance. Between the electrodes, the following boundary condition is applied:

$$\frac{\partial u}{\partial n} = 0 \quad \text{on } \partial \Omega \setminus \{\partial \Omega_1 \cup \ldots \cup \partial \Omega_L\}$$

(4)

which specifies that no current density flows out of the imaged region, except through electrodes. Equations (1-4), together with the charge conservation law:

$$\sum I_l = 0$$

(5)

and the specification of a reference potential:

$$\sum V_l = 0$$

(6)
completely describe the forward problem allowing for solution of the potential distribution $u$ and the voltages at the electrodes $\{V_1 \ldots V_L\}$ given the geometry of the imaging region, the distribution of the conductivity $\sigma$, and the applied currents $\{I_1 \ldots I_L\}$.

The sensitivity of measurements to variations in the conductivity distribution can be computed with the forward model. For tetrapolar measurements, the change in the voltage difference $V_i - V_j$, or sensitivity to a variation in the conductivity distribution, $\delta \sigma$, given that the current was injected through the electrode pair $(m, n)$, is (Geselowitz 1971):

$$\delta (V_i - V_j) = - \int _{\Omega} \delta \sigma \ E_{\text{lead}(i,j)} \cdot E_{\text{applied}(m,n)}$$

(7)

where $E_{\text{applied}(m,n)}$ is the actual electric field resulting from the applied current at electrode pair $(m, n)$, and $E_{\text{lead}(i,j)}$ is the electric field that would result from the application of a unitary current stimulus at measurement pair $(i, j)$. We will use this definition of sensitivity to optimize the tetrapolar measurement patterns we apply with the probe, as discussed in the following sections.

3. Probe Geometry and FEM Models

We intend to acquire concurrent US and EIT imaging data from of the prostate by equipping an endorectal US probe with a set of electrodes. A sonolucent array of electrodes printed on a 1 to 2 mil thick disposable Kapton sleeve will cover the probe and provide coregistered electrical properties and ultrasound imaging. Specifically, we will use a TargetScan 3D US system (Envisioneering Medical Technologies, St. Louis, MO) which consists of a probe with a single US element that is mechanically rotated and translated to construct a full 3D volumetric view of the prostate without moving the probe, itself. This arrangement limits movement artifacts and allows for precise coregistration of the EIT image with the US data because the EIT sensing array is fixed with respect to the probe and therefore with the imaging field-of-view. Geometrically the TRUS probe is approximately 20 cm in length, and 2 cm in diameter, and has a spherical tip at its distal end to facilitate rectal insertion.

The TREIT data acquisition system has been developed in house and is based on a commercial Data Acquisition board and a custom analog front–end that performs the switching of a current source and the voltage sensing terminals on 32 electrode channels. In our design, we intend to place 30 electrodes on the probe surface and use one of the remaining channels as a distal electrode on the body of the patient. On the probe, we arrange the electrodes in a 2D array with 6 horizontal and 5 vertical, for a total of 30 electrodes. The array covers the area of the probe interested by the prostate, extending vertically for 6 cm and horizontally 180 degrees in aperture (see Figures 1 and 3). Given the $6 \times 5$ grid on which they are placed, the electrodes can be made of larger or of smaller areas resulting in smaller or larger inter–electrode gaps, representing a tradeoff: larger electrodes provide better contact but the resulting smaller
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inter–electrode gap leads to larger shunting currents from electrodes not involved in excitation. This shunting ultimately reduces the ability of the probe to sense at distance, as a fraction of the injected current does not travel into the tissue, but has a return path through the electrode array, itself. In order to evaluate the amount of shunting effect, we use a FEM forward solver coupled to the Complete Electrode Model (Somersalo et al. 1992) to study sensitivity and shunting for three electrode designs based on a $6 \times 5$ grid. The array designs, illustrated in scale in Figure 1, have the following dimensions:

- **Electrode array 1**: electrode height = 0.9 cm, electrode width = 0.1 cm, gap between vertically aligned electrodes = 0.32 cm, gap between horizontally aligned electrodes = 0.42 cm. This configuration has the smallest electrode area and is illustrated in subfigure (a) of Figure 1.
- **Electrode array 2**: electrode height = 1 cm, electrode width = 0.2 cm, gap between vertically aligned electrodes = 0.22 cm, gap between horizontally aligned electrodes = 0.32 cm. This configuration has a medium electrode area and is illustrated in subfigure (b) of Figure 1.
- **Electrode array 3**: electrode height = 1.1 cm, electrode width = 0.4 cm, gap between vertically aligned electrodes = 0.12 cm, gap between horizontally aligned electrodes = 0.12 cm. This configuration has the largest electrode area and is illustrated in subfigure (c) of Figure 1.

The freeware program, NetGen (www.hpfem.jku.at/netgen), was used to produce geometrically accurate meshes describing the probe and the three different electrode array configurations. Figure 2 shows a representative full volume mesh. The probe is inserted into a cylindrical body that simulates the presence of tissue surrounding the probe (imaging region). The embedding cylinder has a radius of 12 cm and a height of 20 cm, with the electrode array approximately centered vertically and beginning at a height of 6.8 cm from the bottom. The cylindric body is sufficiently large in diameter and height that currents injected between pairs of electrodes on the array fall off significantly before reaching the mesh boundary (Jossinet, Marry & Montalibet 2002). In this respect, the arrangement is equivalent to having the probe immersed in an infinite medium. Figure 3 shows an enlarged view of the probe tip and electrodes, for the 2nd array design in Figure 1. We controlled the mesh generation so that 1) finer elements are created near the probe surface, and 2) the mesh becomes coarser with increasing distance from the probe. Figure 4 contains a cross–section of the volume mesh which indicates the spatially dependent mesh densities. The number of nodes and elements varied slightly for the meshes generated for the three electrode array configurations (120,000 nodes and 680,000 tetrahedral elements). We used an optimized forward solver developed at Dartmouth College (Borsic, Hartov, Paulsen & Manwaring 2008) to compute and perform the potential distributions the sensitivity analyses. The solver is capable of executing on multicore computing platforms and handling large problems very efficiently.
4. Measurement Pattern Optimization

EIT data acquisition systems can be categorized as adaptive or pair-driven systems. Adaptive systems are able to inject currents through multiple channels simultaneously that can be optimized, in some sense, for estimation of the conductivity distribution. Pair-driven systems are simpler, having only a single current source that injects and sinks current from a single pair of electrodes at any given time. Optimization strategies have been developed for adaptive systems (Isaacson 1986) (Demidenko, Hartov, Soni, Paulsen & Coll 2005) that are extended easily to any number of electrodes. Adjacent and opposite pairs are valid measurement strategies for pair-driven systems which interrogate 2D domains with a set of electrodes located on a circle; however, an optimal measurement strategy for 3D volumes is not as well established. Choices of which quadruplets of electrodes to use for tetrapolar measurements for 3D electrode arrays has been discussed primarily in the context of head imaging (Tidswell, Gibson, Bayford & Holder 2001) (Polydorides, Lionheart & McCann 2002), but no general principles have been described for making these decisions.

Jossinet (Jossinet, Marry & Matias 2002) (Jossinet, Marry & Montalibet 2002) evaluated 2D endotomographic probe with 16 electrodes and proposed a scheme where: 1) voltage sensing pairs were formed from neighboring electrodes which minimizes the difference in the lengths of the leads and therefore the susceptibility to electromagnetic noise, 2) electrodes were placed completely around the probe, 3) “a basic measurement scheme” was formed and then rotated around the probe to acquire all of the measurements, and 4) of the 49 possible excitations pairs within the basic measurement scheme, the 13 linearly independent pairs that resulted in the largest sensitivity in an annular region of interest (ROI) at some distance of the probe were selected. While this method produced good results for a 2D circular probe, we have developed a more general method that is applicable to any 2D / 3D electrode arrangement. The method automatically chooses a full set of linearly independent tetrapolar measurement patterns that maximizes the measurement sensitivity to conductivity changes either across the entire imaging domain or in a specified ROI.

Equation (7) expresses the sensitivity of a voltage measurement between electrode pair \((i, j)\), to a change in conductivity, given that the current is applied through pair \((m, n)\). In our method, we form all possible voltage sense and current drive quadruplets \((m, n, i, j)\) and compute a rank for each of them based on the sensitivity:

\[
\text{rank}(m, n, i, j) = \left| \int_{ROI} E_{\text{lead}(i,j)} \cdot E_{\text{applied}(m,n)} \right| \tag{8}
\]

which is proportional to the sensitivity of the tetrapolar measurement defined by the quadruplet \((m, n, i, j)\) to a uniform change in the ROI. Our goal is to select from all possible quadruplets all the linearly independent measurement patterns, and to do so by examining them in decreasing rank order. In this way, we define a tetrapolar measurement scheme that maximizes the sensitivity to changes in the ROI. The ROI can be taken to be equal to the whole imaging domain or to a subvolume of interest. The
method that we used for testing the linear independence of the measurement quadruplets is based on the generalized impedance matrix (Mitra 1963) defined as:

\[ V = ZI \]  \hspace{1cm} (9)

where \( V \) is the vector of voltages at the electrodes, referenced to an arbitrary external ground potential, and \( I \) is the vector of currents flowing in the electrodes. The voltage, \( V_i \), resulting at the \( i \)-th electrode from the excitation at the pair \((m, n)\) can be expressed in as:

\[ V_i = (Z_{i,m} - Z_{i,n}) I_{\text{injected}(m,n)} \]  \hspace{1cm} (10)

where \( I_{\text{injected}(m,n)} \) is the intensity of the current injected between electrode pair \((m, n)\). The voltage, \( V_j \), resulting at electrode \( j \) from the same excitation is expressed as:

\[ V_j = (Z_{j,m} - Z_{j,n}) I_{\text{injected}(m,n)} \]  \hspace{1cm} (11)

A measured difference of potential, \( V_{\text{meas}(i,j)} \), resulting from the tetrapolar measurement \((m, n, i, j)\) must equal:

\[ V_{\text{meas}(i,j)} = [(Z_{i,m} - Z_{i,n}) - (Z_{j,m} - Z_{j,n})] I_{\text{injected}(m,n)} \]  \hspace{1cm} (12)

and; therefore, the quadruplet \((m, n, i, j)\) establishes a linear relationship involving the \( Z_{i,m}, Z_{i,n}, Z_{j,m}, Z_{j,n} \) elements of \( Z \). Multiple tetrapolar measurements can be used to form a linear system of equations that ultimately determines all unknown elements of \( Z \):

\[ Mz = z_{\text{meas}} \]  \hspace{1cm} (13)

where the matrix, \( M \), contains the relationships established by each measurement quadruplet \((m, n, i, j)\), where \( z \) is the vector containing the elements of the matrix, \( Z \), arranged in column–wise order:

\[ z = [Z_{1,1}, \cdots, Z_{n,1}, Z_{1,2}, \cdots, Z_{n,2}, Z_{1,3}, \cdots, \cdots, Z_{n,n}]^T \]  \hspace{1cm} (14)

and where \( z_{\text{meas}} \) is the vector of the measured apparent impedances \( V_{\text{meas}(i,j)}/I_{\text{injected}(m,n)} \) for each measurement quadruplet \((m, n, i, j)\). Rows of the matrix, \( M \), corresponding to each measurement \((m, n, i, j)\) will have four non–zero entries:

\[ [\cdots, +1_{i,m}, \cdots, -1_{i,n}, \cdots, -1_{j,m}, \cdots, +1_{j,n}, \cdots] \]  \hspace{1cm} (15)

where the subscripts \((i, m)\) indicate the column index in \( M \) that corresponds to the index of \( Z_{i,m} \) in the vector \( z \). A tetrapolar measurement quadruplet can be tested for linear independence from a set of pre–existing tetrapolar quadruplets if the row in \( M \) that corresponds to the new quadruplet is linearly independent from all other rows of \( M \) that correspond to the pre–existing measurement set.

Thus, the measurement optimization scheme that we have developed consists of an algorithm which given an FEM forward mesh:

- Forms all possible measurement quadruplets \((m, n, i, j)\)
• Computes the rank of each quadruplet according to (8)
• Orders the quadruplets in descending rank
• Starting from the quadruplet with the highest rank, forms rows of the matrix, \( M \), and inserts a new quadruplet only if linearly independent from the preceding ones.†
• On termination, the algorithm outputs the set of linearly independent quadruplets.

For \( n \) electrodes, there are \( n^4 \) possible measurement quadruplets, making the algorithm potentially very intensive both in computing the ranks and in selecting the linearly independent measurements from all possible quadruplets. To expedite the calculations, we initially compute a set of forward solutions using a basis of linearly independent excitations. From this solution set, we can quickly produce any new forward solution by superposition. In the formation of the possible excitation and measurement pairs, if pair (1, 2) is selected pair (2, 1) is not selected, as the sensitivity would be the same (just the sign reversed) and pairs (1, 1) and (2, 2) are never chosen. This reduces the total number of pairs to be tested from \( n^4 \) to \( [\text{Fib}(n−1)]^2 \), where Fib is the Fibonacci series, that is from 810,000 to 189,225 quadruplets for a mesh with 30 electrodes. Though the number of quadruplets to be ranked (and tested for linear independence) is large, a MATLAB implementation of our algorithm is able to compute ranks for our meshes in about 10 minutes.

The computing bottleneck in the algorithm is the linear independence test, performed when inserting a new row into matrix \( M \) and evaluating whether the rank has incremented (or not). MATLAB computes the rank of a matrix by SVD decomposition and by counting how many singular values exceed a certain threshold. It is not necessary, however, to test all quadruplets: as soon as the algorithm finds the maximum number of linearly independent tetrapolar measurements, the process can be terminated. In our experience, a complete set of independent measurements is found well at the beginning of the whole quadruplets set, and therefore, the termination condition can shorten significantly the execution time of the algorithm. It can be demonstrated (see Appendix A) that for \( n \) electrodes, for isotropic media, the maximum number of linearly independent measurements is \( \text{Fib}(n−1) \). If the electrodes engaged in current excitation are excluded from the measurements, as is the case in tetrapolar recordings, this number is reduced to \( \text{Fib}(n−1)−n \), or 405 for 30 electrodes. We use this as a termination condition.

5. Pattern Optimization and Sensitivity for the Segmented Prostate

One of the primary issues associated with estimating tissue electrical properties using an electrode array placed inside the body is the rapid decay in sensitivity with increase in distance from the electrode. Jossinet (Jossinet, Marry & Matias 2002) (Jossinet, Marry & Montalibet 2002) has studied a cylindrical probe with 16 electrodes equispaced around

† We consider any condition of \( M \), such as symmetry, that might make a new measurement dependent on previous ones – please see Appendix A
its circumference and shown experimentally that the image field of view is approximately 3 times the diameter of probe despite the fast decay in sensitivity. The intended application for Jossinet's configuration was transurethral prostate imaging which would limit the diameter of the electrode array to be less than 8 mm creating a total imageable volume of a cylindrical region 24 mm in diameter. The typical prostate has a diameter of 50 mm exceeding the limits of this type of probe. Our aim is to couple US and EIT in a configuration that is able to sense the entire prostate. The combined use of US imaging will allow us to delineate boundaries of the prostate and to use this prior structural information in the estimation of the electrical properties of the tissues. Prior information can be incorporated in EIT algorithms in a number of ways including soft–priors (Borsic, Lionheart & McLeod 2002) (Kaipio, Kolehmainen, Vauhkonen & Somersalo 1999) and hard–priors (Vauhkonen, Kaipio, Somersalo & Karjalainen 1997). Soft priors are usually incorporated into the regularization term in image reconstruction algorithms. Hard priors can be implemented by segmenting the imaging domain into a number of regions and constraining the conductivity to be uniform within each zone. The choice between soft priors and hard priors generally depends on the reliability of the prior information that is available. Hard priors prove better where accurate information regarding the location of boundaries is known. In the case of coregistered US, EIT prostate imaging, the boundaries of the prostate can be accurately extracted from US imaging. The extracted volume can be subdivided in a small number of voxels, for which the conductivity can be estimated via EIT. We will use thus a hard–priors approach, assuming the conductivity as constant in these voxel regions, and formulating the inverse problem as the estimation of these few parameters.

In this section, we optimize measurement sensitivity for estimation of electrical properties in regions of the prostate that will be segmented. The prostate is simulated by embedding an ellipsoid into the FEM mesh with axial dimension 50 mm, an anteriorposterior dimension of 40 mm and a transverse dimension of 50 mm, which are representative of normal prostate (Özden, Turgut, Talas, Önder & O. 2007). The simulated prostate is placed 5 mm from the EIT imaging array, as illustrated in Figure 5 which is representative of transrectal imaging, where the US and EIT sensors are separated from the prostate by the rectal wall. The prostate is segmented in 12 voxels; the volume is divided by two ±45° planes passing through the axial axis and further subdivided into in three levels along the axial dimension. The checkerboard coloring of the simulated prostate in Figure 5 corresponds to this subdivision of its volume. Figure 6 illustrates the numbering of the voxels in which the prostate has been subdivided and for which we estimate the electrical conductivity. The figure shows cross–sections of the simulated prostate in the transverse plane intersecting the bottom, middle and top layer. In each subfigure, the voxels for each layer are displayed along with their numbering. The small circle below the prostate cross-section represents the position of the US EIT probe.

Voxels 2, 6 and 10 are the farthest from the probe, being on the opposite side of the prostate. For these voxels, EIT sensitivity is expected to be the smallest, as most
of the current injected between pairs of electrodes will flow in proximity of the probe surface. Jossinet (Jossinet, Marry & Matias 2002) (Jossinet, Marry & Montalibet 2002) showed the rapid decay in sensitivity with radial distance from the sensing probe in 2D. In the 3D problem, we expect the decay in sensitivity to be even faster as currents spread in three dimensions. Based on these considerations, the most difficult voxels to sense would be numbers 2, 6 and 10. We computed the tetrapolar measurement patterns using the algorithm described in Section 4 to optimize the sensitivity in the ROI formed by the union of the three voxels 2, 6 and 10, or the prostate quadrant opposite to the sensing array. Measurement patterns were computed for a uniform conductivity distribution of 0.1 \( \text{S} \text{m}^{-1} \). With 30 electrodes, we obtained 405 linearly independent patterns. Table 1 reports the first 10 and the last 10 computed tetrapolar measurement patterns, respectively, while Figure 7 shows the numbering of the electrodes on the sensing array. The first, and the most sensitive pattern, is (3, 27, 4, 28). Electrode 3 is located on the bottom row and electrode 27 on the top row of the array (see Figure 7); the current injection between this pair provides a pathway for current flow that reaches a maximum distance from the probe surface since these electrodes are positioned at opposite ends of the array. The same consideration applies for measurement pair (4, 28): the two electrodes represent the maximum separation and give rise to the maximum lead field in the ROI. All of the patterns in Table 1(a) have a large physical separation between the two electrodes involved in current injection and between each of the two electrodes used for voltage sensing resulting in higher sensitivities in the ROI. Table 1(b) shows the least sensitive patterns. Even the least sensitive of the optimal patterns, (17, 26, 10, 14), has an electrode separation in the injection and sensing pairs that allows sensing to a certain distance from the array.

Given the optimized measurement scheme, we computed the sensitivity of the 12 voxels subdividing the prostate for the three different electrode arrays. For these
simulations we used a uniform conductivity of $0.1 \text{ S m}^{-1}$, for the background and for the prostate which is representative of normal tissues (Halter, Schned, Heaney, Hartov, Schutz & Paulsen 2008). We injected current of 1 mA which is the maximum Patient Auxiliary Current allowed by the IEC 601 Standard at 10 KHz, the frequency we intend to employ for data acquisition; we used an electrode contact impedance of $2.35 \times 10^{-4} \text{ \Omega m}^2$, which in our experience fits well EIT data from tank experiments containing physiological saline solution. With these values, we computed the sensitivity for a 10% change in conductivity of each prostate voxel as in (7), using all patterns in the measurement scheme. As expected, the sensitivity varied depending on which electrode array was used, being on average lower for the second and third electrode array designs. These arrays have smaller inter–electrode gaps which result in a larger shunting of the applied currents. The sensitivities for the three electrode array designs, averaged across the 405 tetrapolar measurements, are reported for each prostate voxel in Table 2 and expressed in mV. These are the expected mean changes across all measurements for a 10% change in the conductivity of a voxel, under the specified conditions. We report the mean change across all the measurements as a single figure of merit that summarizes the sensitivity.

Voxels 2 and 10 have the lowest sensitivity as they are on the distal ends of the anterior of the prostate (see Figure 6), where the least amount of current flows. They are followed by voxel 6 which is on the anterior side of the prostate but on the central layer where more current passes. The voxel presenting the highest sensitivity is number 8, which is on the central layer proximal to the EIT electrode array. Overall, the sensitivities are in a range that is detectable by typical EIT instrumentation, and the ratio between the smallest and highest sensitivity within the 12 voxels is approximately 40, which makes the problem of estimating the 12 different voxels conductivities from electrode measurements only mildly ill-conditioned. This inverse problem is much better posed than the typical unconstrained image reconstruction problem in EIT, where without prior segmentation of the imaging volume, the sensitivity across the entire image domain can span 20 orders of magnitude. The sensitivity analysis performed here suggests that with segmentation of the prostate, using prior US information, the electrical properties within the subdivided prostate voxels can be estimated accurately.

Given that all three electrode designs present sufficient sensitivities, the numerical results in Table 2 indicate the significance of the effect through the passive electrodes, and how it is affected by the inter–electrode gaps. Looking, for example, at voxels 2 and 10, for which the sensitivity is the smallest, we see a drop in sensitivity from 0.108 mV to 0.100 mV, or 8% going from electrode array 1 to electrode array 2, and a drop from 0.100 mV to 0.074 mV, or 26%, going from electrode array 2 to electrode array 3. We expect that electrode array 2 might offer a better tradeoff in electrode area versus sensitivity compared to electrode array 3 in practice. To reinforce these findings, we computed the amount of current lost in shunting for the three array configurations. Results are reported in Table 3. We conducted two numerical experiments, computing the shunting current for a vertical excitation pattern and for a horizontal excitation
Table 2. Mean sensitivities for the 12 voxels of the segmented prostate

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<th>Voxel</th>
<th>(a) Electrode Array 1</th>
<th>Voxel</th>
<th>(b) Electrode Array 2</th>
<th>Voxel</th>
<th>(c) Electrode Array 3</th>
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pattern (i.e. current injection pairs (3, 28) and (13, 18), respectively. Results differ slightly for the two directions as the electrodes and inter–electrode gaps are different in the horizontal and vertical directions. As expected, electrode array 2 and 3 result in higher shunting currents than electrode array 1, because these arrays have smaller inter–electrode gaps. Table 3(a) results are relative to a 1mA current excitation between electrode 3, on the bottom row, and electrode 28 on the top row (see Figure 7 for electrode numbering). Because the excitation is along the vertical axis, we computed the total shunting currents passing through a horizontal row of electrodes. In Table 3(a) we report the sum of currents shunted by electrodes 19,20,21,22,23 and 24. As this horizontal row of electrodes intercepts all paths of the shunting currents (flowing vertically), the figure is representative of the total amount of current lost in the array due to the shunting caused by the passive electrodes not involved in the excitation. For array 1, approximately 5% of the injected current (1 mA) is lost, for electrode arrays 2 and 3, the amount is 11%, and 36%, respectively. Electrode array 3 exhibits a very significant shunting effect, which leads to its significant reduction in sensitivity. Similar considerations apply to Table 3(b), where results are relative to a horizontal excitation of 1mA between electrodes 13 and 18, with shunting currents reported for the vertical row encompassing electrodes 2,8,14,20 and 26. These results confirm that the reduction in sensitivity of electrode arrays 2 and 3 over electrode array 1 is caused by the smaller inter–electrode gaps, resulting in larger current shunting.

6. Pattern Optimization and Sensitivity with a Distal Electrode

Even though analysis from the previous sections shows that the 12 voxels in which we segmented the prostate present sensitivities that are detectable with common EIT instrumentation, one way to improve sensitivity to the prostate voxels far from the probe would be to include a distal electrode. In this section we investigate the effect of
Table 3. Shunting currents for a vertical and for an horizontal excitation

<table>
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<tr>
<th>Elec. Array</th>
<th>Shunted Current mA</th>
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<td>1</td>
<td>0.057</td>
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<td>2</td>
<td>0.362</td>
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Table 3. Shunting currents for a vertical and for an horizontal excitation

introducing a such an electrode. In practice, this electrode would be placed on externally on the body of the subject. Current patterns including this electrode will be dependent on the general anatomy of the subject. Patient specific modeling of the full abdomen with a FEM mesh would be impractical though, and we expect to truncate the FEM mesh in a region around the prostate. The use of a distal electrode introduces therefore some modeling errors, as the current paths in the abdominal region are not completely captured by the forward model. We expect to reduce these errors/uncertainties by using the distal electrode only for passing current and by sensing only with electrodes on the probe and not between the probe and the distal electrode. The lead fields resulting from sensing in this way decay rapidly with distance from the probe, and therefore, measurements should be relatively insensitive to model errors distant from the probe.

We modeled the distal electrode as a large 4 cm diameter electrode, placed on the exterior of the mesh, in line with the electrode array, as illustrated in Figure 8. We computed the optimized measurement patterns, including the 31st electrode, using it only for current injection (discarding any measurement pair \((i,j)\) involving it). This leads to a set of 434 linearly independent tetrapolar measurement patterns. In these simulations only electrode array 2 is considered, as it presents a good compromise between electrode area and sensitivity. The sensitivities, computed including the distal electrode, present an average improvement on the order of 5%. There are, however, a few patterns for which the sensitivity is improved considerably. For example, in prostate voxel 2, for which the sensitivity is the lowest (as for voxel 10), we have obtained 15 new patterns for which the sensitivity has largely improved. Figure 9 shows the sensitivity of voxel 2 for all the patterns. The light dotted line indicates the sensitivity for the optimized patterns without the use of the distal electrode while the heavy dotted line reflects the sensitivity with the use of the distal electrode. The maximum sensitivity has improved from 1.7 mV to 2.1. Table 4 reports the 15 patterns for which the sensitivity in voxel 2 has increased over the measurements patterns not involving the distal electrode (numbered 31). It is interesting to note that the injection electrode on the probe side is often electrode number 27,28 or 22 or 21. These are electrodes located on the top row of the probe (see Figure 7) and close to the prostate. The sensing pairs are formed in such a way to present a large distance between the two sensing electrodes, resulting in a far sensing lead field. The sensing pairs often involve electrodes 1 and 6, which are located on the bottom left and bottom right of the array, respectively. The other sensing electrode is often electrode number 21 or 22 which is on the higher rows of
Table 4. Patterns involving the distal electrode which resulted in a sensitivity gain

<table>
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<tr>
<th>m</th>
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<th>i</th>
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</thead>
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</table>

the probe, presenting a large separation from 1 and 6, and therefore, a good sensing capability. Overall, the addition of a distal electrode results in a number of measurement patterns for which the sensitivity is improved which might provide a better quantitative estimation of the tissue electrical properties. Whether the use of this electrode will be effective in the clinical application it is not known at this time, since, as discussed, measurements involving this electrode might be affected by modeling errors.

7. Conclusions

Establishing how electrodes are configured for the US EIT transrectal probe described here is critical to optimizing EIT sensitivity. We elected to employ a rectangular array of 30 electrodes, co-aligned with the acoustic window of the transrectal US probe, in order to provide 1) sufficient area for tissue electrode interface, 2) a number of electrodes within the capabilities of typical EIT systems, and 3) the ability to utilize coregistered US images for enhancing EIT image reconstruction. The electrodes were configured to extend 6 cm along the length of the US probe and to wrap 180° around the shaft. We also consider the presence of an additional distal electrode placed externally to the body of the patient. As these electrode configurations are three dimensional in space and relatively complex, the selection of appropriate measurement patterns is critical. We developed a novel and general method for automatically computing a full set of linearly independent tetrapolar measurement patterns. This method maximizes the sensitivity in a ROI of choice and can be useful in other applications of EIT, particularly in 3D (i.e. brain imaging, breast imaging), where, to our knowledge, no other systematic
method has been established for selecting appropriate tetrapolar measurements. This method can also be extended to other forms of tomography, for example to capacitive sensing and to microwave sensing. The test for linear independence of measurement patterns extends to these applications by extension of the impedance matrix concept. The ranking of measurement quadruplets can be extended as the sensitivity in these methods is expressed by a volume integral of cross products of an applied and a sensing field, similarly to EIT.

Having developed an algorithm for forming appropriate measurement patterns, we study the sensitivity of the resulting measurements to changes in the prostate conductivity values. One of the main difficulties in sensing electrical properties of tissues from the inside is the rapid decay of sensitivity with increasing distances from the sensing probe. In order to partially overcome this difficulty we propose to use prior information from US imaging into EIT reconstruction / parameter estimation. Specifically we intend to delineate the boundaries of the prostate from US, to subdivide the volume in a small number of voxels and to estimate the conductivity as constant on each of these subvolumes. In the simulations we subdivided the prostate in 12 voxels and studied the sensitivity to conductivity changes in such regions. Measurement patterns were computed for optimizing the sensitivity in the distal region of the prostate, where the lowest sensitivity is expected. The sensitivity was studied for three different array designs, presenting different areas and different inter-electrode gaps. For all the three designs the average sensitivity in the 12 voxels results in voltage variations that are detectable by common laboratory EIT instrumentation. We show however that the design with larger electrodes results in a significantly decreased sensitivity, due to large shunting effects arising from smaller inter–electrode gaps. In terms of tradeoff between electrode area and sensitivity array 2 results to be a better choice, as it presents an only slightly decreased sensitivity with respect to array 1 but a larger area. The use of a distal electrode, external to the body of the patient, for injecting currents and thus improving the sensitivity was studied, and this configuration is shown to present few measurements with a significantly larger sensitivity. The use of a distal electrode has the potential to improve the detection capabilities of the proposed imaging configuration, though the clinical effectiveness needs to be studied, as the current paths between the distal electrode and the probe are not captured by the FEM mesh. Sensing only between electrodes on the probe, and never between the distal electrode and the array on the probe, might limit the effect of these modeling errors, as the resulting lead fields (and thus sensitivity) are strong only in the very proximity of the probe array. This approach, which limits the effects of modeling errors, might allow the reconstruction scheme to benefit from the use of this additional electrode.

On the basis of the present analysis we have developed an understanding of the effect of different electrode designs in terms of sensitivity. We believe that the proposed combined US EIT imaging approach can lead to a robust estimation of electrical properties of the prostate, and that it can be useful in the clinical application. We will conduct further numerical simulations using anatomical finite element meshes of the full
abdomen, deriving from MRI images, for studying the use of the distal electrode and the errors arising from mesh truncation in greater detail.

Appendix A. Number of Independent Linear Measurements

This appendix discusses briefly how the number of independent measurements is determined for an $n$-pole. Given an object with $n$ electrodes attached, an $n$-pole, this can be completely characterized by the associated generalized impedance matrix $Z$. As shown in Section 4, different measurement quadruplets $(m, n, i, j)$ allow to establish a linear relationship (12) between elements of $Z$. Multiple measurements can be used to fully determine $Z$ through (13). The maximum number of linearly independent measurements that can be carried on an $n$-pole is equal to the number of independent elements in the associated impedance matrix, as these characterize completely the $n$-pole. Not all the elements of $Z$ are independent: in the generalized impedance matrix of an $n$-pole the sum over rows and over columns is zero (Mitra 1963) and, for isotropic media, the matrix is symmetric. The total number of independent elements in $Z$ can be counted as follows:

The total number of elements in $Z$ is equal to the number of elements on the diagonal, $n$, plus the number of elements on the upper triangular, $\text{Fib}(n-1)$, plus the number of elements on the lower diagonal, $\text{Fib}(n-1)$. In total $n + 2 \text{Fib}(n-1)$. This number must be decreased by the number of conditions that establish dependance between the elements of $Z$ in order to find the number of independent elements.

- in the isotropic case, where $Z$ is symmetric, the upper and the lower triangular matrices are identical, this establishes $\text{Fib}(n-1)$ equality conditions between corresponding elements in the upper and in the lower triangular matrices, reducing the total number of independent elements to $n + \text{Fib}(n-1)$.
- the zero sum over rows condition establishes other $n$ dependance conditions reducing the number of independent elements to $\text{Fib}(n-1)$
- the zero sum over columns does not reduce any further the number of independent elements as this is accounted for by the symmetry and zero sum over rows conditions automatically.
- if sensing electrodes are constrained to be different from injecting electrodes, this means the in the measurement quadruplets $(m, n, i, j)$ we have always $i \neq m$, $i \neq n$, $j \neq m$, $j \neq n$. This implies by inspection of (12) that there is no access to the diagonal elements of $Z$. This reduces further by $n$ the number of elements in $Z$ that can be determined, this resulting equal to $\text{Fib}(n-1) - n$

References


Figure 1. *Three electrode arrays printed on a flat Kapton sheet, resulting in different inter–electrode gaps.*

Figure 2. *Wireframe representation of the probe mesh.*

Figure 3. Detail of the electrode array of the probe mesh.

Figure 4. Cross section of the volume mesh showing a decreasing mesh density with distance from the probe.

Figure 5. Illustration of the simulated prostate showing in checkerboard colors the subdivision of the prostate in voxels.
Figure 6. Numbering of the voxels for the segmented prostate.

Figure 7. Electrode numbering for the electrode arrays.
Figure 8. Wireframe representation of the probe mesh showing the distal electrode on the outer surface of the mesh.
Figure 9. Plot of the sensitivity with and without the distal electrode. The measurement sensitivity to a 10% conductivity change is plotted for voxel 2, which (with voxel 10) is the most difficult voxel to sense as it lies on the far side of the prostate and on the top layer. The dotted line represents the sensitivity without the distal electrode, plotted in decreasing order for all measurement patterns. The continuous line represents the sensitivity for voxel 2 with the distal electrode, plotted in decreasing order for all measurement patterns. The sensitivity units are mV.