Dynamics of the Oceanic Surface Mixed Layer

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SUBDUCTION

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ABSTRACT

Subduction, or the escape of fluid from the mixed layer, and its counterpart, entrainment of underlying stratum, are processes that couple the dynamics of both the mixed layer and the ocean interior. However, it is customary for mixed-layer dynamicists to take the state of the ocean interior for granted, while large-scale circulation theorists have a record for brushing aside the mixed layer as a mere converter of surface wind stress into vertical Ekman pumping. As the recent progress in the theory of large-scale ocean circulation has revealed the crucial role played by subduction in maintaining the permanent thermocline, and as the corresponding models become increasingly refined, the process of subduction deserves timely consideration.

The purpose of the present introductory analysis is to elucidate the kinematics and dynamics of subduction (and of its twin process, entrainment) by inductive reasoning through a series of increasingly more comprehensive models. Some likely pitfalls (e.g. the common belief that the rate of subduction is the Ekman pumping velocity) are identified, and several relevant dimensionless numbers are noted.

Particular attention is directed toward seasonal variations of the mid-latitude mixed layer, which control the intermittency, amount, density and potential vorticity of the subducted waters.

INTRODUCTION

Definition of subduction

Because subduction is such a new buzz word in physical and chemical oceanography, a definition seems prerequisite to any further discussion. Because of its generality, I propose the following definition: Subduction is the process by which mixed-layer convergence and/or retreat leave formerly turbulent fluid to become part of the underlying stratum.

Note that this definition does not make a distinction between diurnal and seasonal mixed layers, nor does it specify the underlying stratum except that it be markedly less turbulent than the mixed layer. Subduction from the diurnal wind-mixed layer during nighttime restratification can thus feed the seasonal thermocline, while subduction induced by the retreat of the winter convective layer imparts fluid for the permanent thermocline and the large-scale circulation. Note also that no distinction is made between ventilation and subduction. In a layered model (as that of Luyten et al., 1983), ventilation is the fluid flow from the mixed layer into a layer of the stratum while subduction is the submersion of that layer under the next lighter one. But, for the continuously stratified ocean, this distinction is superfluous, and ventilation and subduction are equivalent.

Definition of subduction rate

As for many physical concepts, the same word can be used both to describe a process and to quantify it. The considerations developed later on make it natural to choose the following definition:

The subduction rate is the flux of volume of subducted fluid per unit horizontal area. It is therefore expressed in meters per second. This rate is chosen as negative if the flux is in the direction of subduction and positive if the flux is in the direction of entrainment.

Although the subduction rate connotes the idea of a vertical velocity, it is not a vertical velocity as the next considerations demonstrate.

VERTICAL VELOCITIES AND VOLUME FLUXES

The vertical excursion of the mixed-layer base, the vertical velocity of the fluid at that level, the Ekman pumping, and the subduction rate all seem to be related but are obviously not equivalent. A clarification is therefore most helpful.

As sketched on Figure 1, consider a vertically homogeneous mixed layer and, inside it, an infinitesimal fluid column of unit horizontal cross section. Since incompressibility of seawater is negligible, the mass budget of this fluid element can be stated as a volume budget. Leaving fluid comprises evaporation at the surface (Ev), lateral outflow and subduction through the mixed-layer base (Su, if Su < 0). Entering fluid includes precipitation at the surface (Pr), lateral inflow and entrainment (Su, if Su > 0). The net difference between inflows and outflows goes into storage or deepening of the mixed layer ($\partial h/\partial t$). The volume budget is therefore

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = Su + Pr - Ev. \qquad (1)$$

Precipitation minus evaporation is considerably less than the other fluxes (less than a meter per year versus several tens of meters per year), and the term Pr–Ev is heretofore neglected. Note that writing simply Su (and not Su multiplied by the cosine of a projecting angle) on the right of (1) required that the subduction rate be measured per unit horizontal area. This justifies the careful definition of the subduction rate in the previous section.

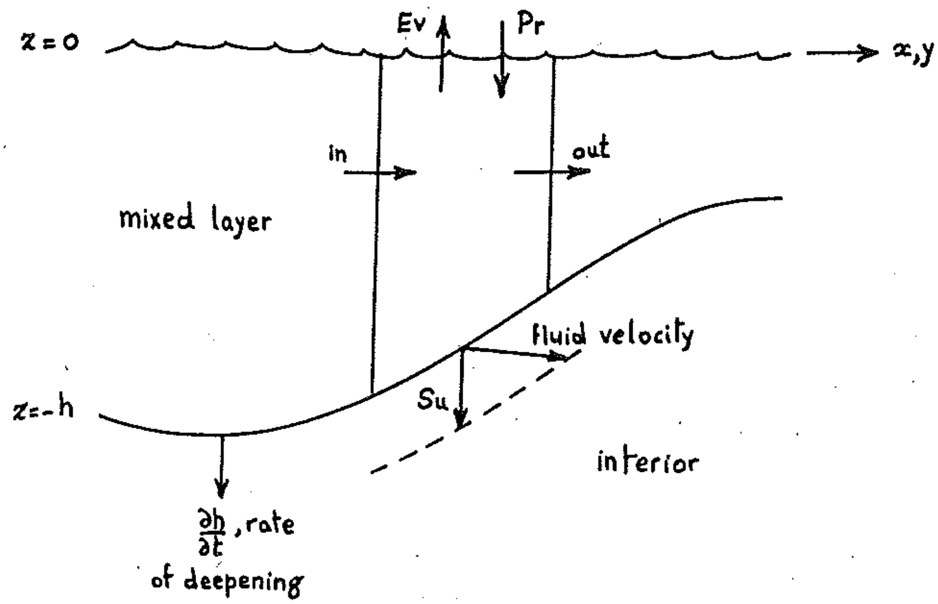

Figure 1. Sketch of the principal velocities and volume fluxes.

The vertical velocity of the fluid at the base of the mixed layer is given by

$$w = -\left[\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y}\right] + Su. \tag{2}$$

The bracketed quantity would be the vertical velocity in the absence of subduction or entrainment (fluid flowing along the mixed-layer base, $z = -h$), while the last term would be the vertical velocity if the mixed-layer base were horizontal and stationary. There is thus a relation between vertical velocity (w), mixed-layer deepening ($\partial h/\partial t$) and subduction rate (Su). Combining (1) and (2), the subduction rate can be eliminated to obtain

$$w = h\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right). \tag{3}$$

If one now makes the <u>assumption</u> that the horizontal velocity (u,v) in the mixed layer is only the Ekman drift ($\tau^y/\rho_0 fh$, $-\tau^x/\rho_0 fh$), the former relation (1) for Su becomes

$$Su = \frac{\partial h}{\partial t} + Ek, \tag{4}$$

where $Ek = \partial(\tau^y/\rho_0 f)/\partial x - \partial(\tau^x/\rho_0 f)/\partial y$ is the Ekman pumping. Averaging over a mixed-layer cycle (a day or a year), one finally obtains

$$\overline{Su} = \overline{Ek}, \tag{5}$$

where an overbar denotes a suitable time average. It is therefore concluded that the subduction rate is in general not the Ekman pumping, and that it is the Ekman pumping only if (i) precipitation and evaporation rates are negligible, (ii) the mixed-layer flow is very close to being the Ekman drift, and (iii) a time average is performed over the mixed-layer cycle.

Although a time average of relation (4), which is linear, is straightforward, it must be kept in mind that other quantities such as density and potential vorticity will not in general be linearly related to mixed-layer variables and, therefore, that the time averages of these quantities over a mixed-layer cycle will not be equal to the same quantities calculated from a time-averaged mixed-layer model. This is a very important problem, which motivates the careful evaluation of the time-dependent models presented in the subsequent sections.

The assumption of a mixed-layer flow dominated by Ekman dynamics is widely used in large-scale oceanography studies and thus deserves some consideration. In a low Rossby-number regime, relative acceleration and nonlinear advection can be safely neglected but a geostrophic current can be important. Let us write

$$-\rho_0 f v = -\frac{\partial p}{\partial x} + \frac{\tau^x}{h}, \quad \rho_0 f u = -\frac{\partial p}{\partial y} + \frac{\tau^y}{h},$$

where $f = f_0 + \beta y$ varies with latitude and p is the mixed-layer pressure comprising the pressure in the underlying flow field (dynamic topography) and the pressure distribution resulting from an uneven mixed-layer base. The divergence terms of (1) now become

$$\frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = Ek + J\left(\frac{p}{\rho_0}, \frac{h}{f}\right), \tag{6}$$

where $J(a,b) = a_x b_y - a_y b_x$ is the Jacobian operator. The last term is new and can be split into two parts: the Jacobian of the pressure p with the mixed-layer depth h, which is not zero if the isolines of p do not follow those of h, and the Jacobian of the pressure p with the Coriolis factor f. Separating among these two contributions and implementing (6) in (1) to evaluate the various contributions to subduction, one finds

$$Su = \frac{\partial h}{\partial t} + Ek + \frac{1}{\rho_0 f} J(p, h) - \frac{\beta h}{\rho_0 f^2} \frac{\partial p}{\partial x}. \tag{7}$$

Physically, the four terms can be interpreted as follows: the first term represents the subduction caused by mixed–layer retreat; the second, subduction by Ekman pumping injection; the third, subduction resulting from convergence of a geostrophic flow squeezed into a shallower mixed layer; and the fourth, subduction caused by a decelerating poleward geostrophic flow on a beta plane. Orders of magnitude for these various contributions can be estimated with the following typical numbers:

variations of $h \sim 50$ m in 500 km and in one year,
$Ek \sim 30$ meters per year,
geostrophic flow ~ 1 cm/s,
$f \sim 10^{-4}s^{-1}$, $\beta \sim 2\ 10^{-11}m^{-1}s^{-1}$ at mid latitudes.

One finds, in meters per second,

$$\frac{\partial h}{\partial t} \sim 10^{-6}, \quad Ek \sim 10^{-6}, \quad \frac{1}{\rho_0 f} J(p,h) \sim 10^{-6}, \quad \frac{\beta h}{\rho_0 f^2}\frac{\partial p}{\partial x} \sim 2 \cdot 10^{-7}.$$

In conclusion, it must be recognized that geostrophic currents in the mixed layer can contribute to subduction as much as the Ekman pumping does, and that the neglect of convergence of geostrophic currents in the mixed layer is but a regrettable assumption of modelling convenience.

DEPTH–TIME MODEL

In addition to the important distinctions reviewed in the preceding sections, complexity arises because subduction is an intermittent process. Indeed, mixed layers are extremely variable in time, to the point of being classified by the length of their cycles (diurnal mixed layer and seasonal thermocline). Here, attention will be restricted to seasonal variations, for the author is concerned with the input of waters into the permanent thermocline.

A portion of the waters left behind as the mixed layer retreats (typically, from March to August) may not have penetrated sufficiently deeply by the time the mixed layer deepens again (typically, from September to February), and so may be recaptured to participate once more in the mixing process. Intermittency thus arises, for only a portion of the annual cycle is producing subduction and, moreover, only a portion of this subduction phase is not in vain. To determine the amount and type of subducted waters, it is therefore imperative to distinguish clearly between the following three phases: effective subduction (waters are subducted that will not be recaptured), temporary subduction (waters are subducted that will be recaptured), and entrainment (recapture of previously subducted waters).

To go further, it is most helpful to base our considerations on a simple example. Take the case of Stommel (1979): location 25°N–35°W in the central North Atlantic and its vicinity (the so-called Beta Triangle). There, the mixed layer has an annual cycle with a maximum depth of 100 meters that is reached around March 1. After summer restratification, the mixed layer is absent (depth = 0). At the same location, the vertical velocity, evaluated by the beta–spiral method, falls between –1.0 and –1.5 10^{-6} m/s. The Ekman pumping is downward, and its annual average is estimated around 0.94–0.98 10^{-6} m/s. Therefore, a plausible value for the vertical velocity in the interior is 1×10^{-6} m/s or 30 meters per year, in the downward direction. A simple model can then be devised with the mixed-layer depth following $h(t) = 50 + 50 \cos (2\pi t)$ and a parcel's trajectory in the geostrophic interior following $z(t) = z_0-30t$, where the depth unit is the meter, time is counted in years ($t = 0$ is March 1, $t = 1$ is March 1 one year later), and z_0 is a constant that varies from parcel to parcel depending on its depth and time of origin.

Here, the seasonal variation in the vertical velocity is neglected, and so are the lateral variations in the mixed-layer properties and interior flow. Relaxation of some of these simplifications is the object of the next two sections, and the simple depth-time model is meant only as an illustrative support for key ideas and as a first step of an inductive approach.

Figure 2 displays the relevant curves in the depth-time framework: the sine curve represents the mixed-layer depth while the sloping straight lines are various trajectories of the interior flow. One such trajectory is particular: it is the one that is tangent to the sine curve, in the vicinity of $t = 0$. Another similar trajectory is tangent in the vicinity of $t = 1$. Each trajectory corresponds to the same particular parcel, but at a one-year interval. The corresponding parcel is the last one to penetrate into the interior for that year by successfully escaping the mixed-layer deepening in late winter. The subsequent parcels will indeed be recaptured. The time t_0 (or t_0+1), where tangency occurs, is the time of year when the mixed layer ceases to recapture previously subducted waters and when effective subduction begins. For the expressions stated above, the curves are tangent at $t_0 = -0.015$ ($t_0+1 = 0.985$), that is day 359 of the year or February 22. The mixed-layer depth is then 99.8 meters, almost its maximum value. Therefore, February 22 is the time of year when active subduction starts, although the mixed layer still deepens until March 1, but at a rate slower than the downwelling rate of the interior. From February 22 until March 1, the mixed layer is no longer able to overtake descending parcels.

The parcel whose trajectory hugs the mixed-layer curve at t_0+1 has $z_0 = -70.23$ and its trajectory is $z(t) = -70.23-30t$. Tracing this trajectory backward brings it to the mixed-layer curve at time $t_1 = 0.166$, that is day 61 of the year or April 30. This is the time of year when that last parcel to make it successfully into the interior is subducted. After April 30, the mixed layer continues to retreat, but the parcels that it releases will be recaptured the following fall or winter. Therefore, April 30 is the time of year when effective subduction ends and temporary subduction begins. The period of active subduction thus lasts over two months of the year, from February 22 to April 30, that is late winter and early spring. Subduction cannot be said to be as intermittent as the shutter of a camera.

Finally, the time t_2, when the difference between $|z(t)|$ of our particular parcel and $h(t)$ is the largest, is the time of year when temporary subduction ends and when recapture of previously subducted waters begins. The separation between the two curves represents the amount of water that has been subducted in vain. Numbers provide: $t_2 = 0.515$, day 188 or September 4, and $|z(t_2)| - h(t_2) = 85.5$ meters.

Overall, through the year, 30 meters of water have been effectively subducted from February 22 to April 30, and 85.5 meters have been subducted from April 30 to September 4 but only to be remixed during the period from September 4 to February 22. The efficiency of subduction can be evaluated both in time and in volume:

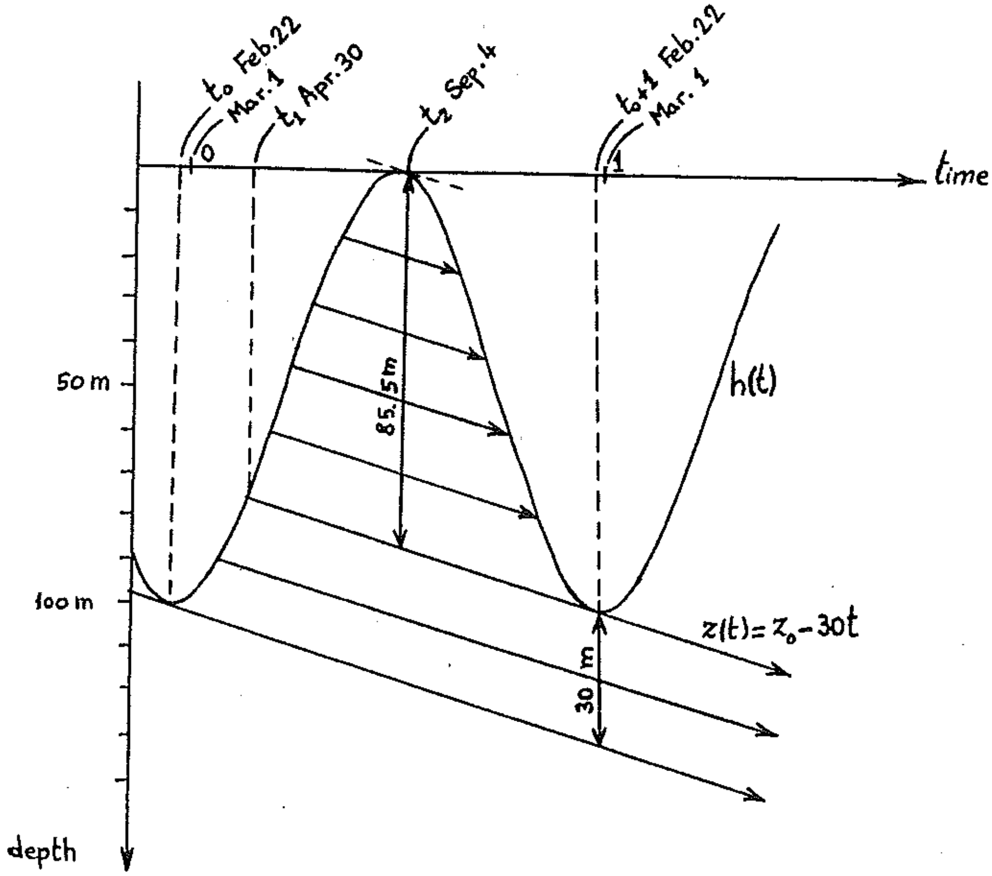

Figure 2. Schematic diagram of subduction according to a simple depth–time model

$$\text{Efficiency in time} = \frac{\text{duration of effective subduction}}{\text{duration of subduction}} = \frac{t_1 - t_0}{t_2 - t_0} = 34\%, \qquad (8)$$

$$\text{Efficiency in volume} = \frac{\text{volume effectively subducted per year}}{\text{total volume subducted per year}}$$

$$= \frac{30\text{m}}{30\text{m} + 85.5\text{m}} = 26\% \qquad (9)$$

In conclusion, subduction is not as efficient as an engineer would expect from one of his/her machines, but is not as stroboscopic as one oceanographer could have anticipated.

Various cases can now be explored. Figure 3 depicts the subduction scenarios in three cases: weak downward flow, large downward flow, and upward flow. In the first case, subduction is very brief and inefficient; in the second case, it occurs throughout

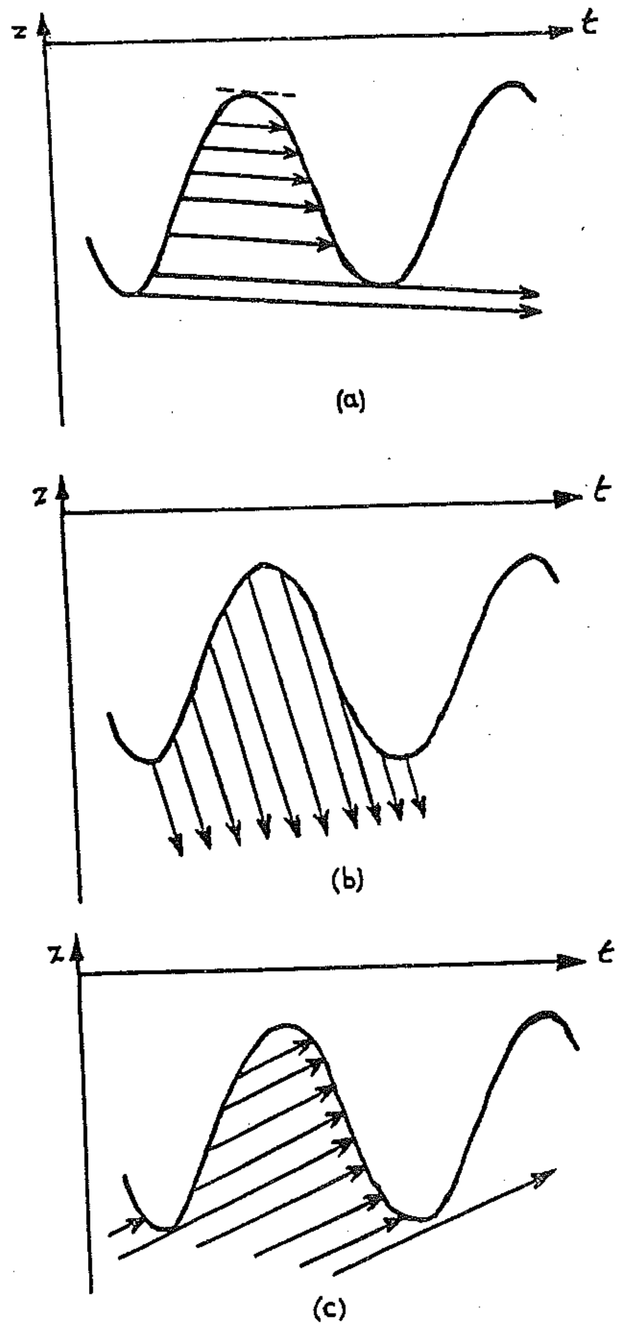

Figure 3. Various subduction regimes. (a) Interior vertical velocity downward and small; efficiency of subduction is low. (b) Interior vertical velocity downward and large; effective subduction occurs continuously. (c) Interior vertical velocity upward and not too large; no effective subduction occurs, but temporary subduction takes place.

the seasonal cycle; and in the third case, although there is no effective subduction, there remains the possibility of temporary subduction. From this range of possibilities, two important factors emerge: (i) the sign of the vertical velocity in the interior (remember, it is not necessarily the Ekman-pumping rate!), and (ii) the ratio of vertical velocity to maximum rate of deepening. The sign controls the existence of effective subduction (positive velocity means no effective subduction), while the ratio controls the efficiency of subduction (ratio less than unity means only partly efficient subduction, ratio greater than unity means 100% efficiency). From the orders of magnitude provided in the previous sections, one notices that the ratio will in general be in the vicinity of unity.

LATITUDE-DEPTH-TIME MODEL

Although illustrative, the above model has serious shortcomings. Perhaps chief of all is its prescription for the density profile in the interior. Indeed, the density varies in the mixed layer with the seasons so that the annual batch of subducted waters is somewhat stratified, but the same layering repeats year after year, and denser waters will top lighter waters. Therefore, under the above scenario, vertical mixing must occur, and the net effect is the formation of a homogeneous water column in the interior. This certainly is not the case for the ocean, and the model must be extended.

Vertical stratification can be explained (Luyten et al., 1983) by the higher latitudes of origin of the deeper waters. In the Subtropical Gyre where the Ekman pumping is downward, the meridional velocity is equatorward. Therefore, the deeper the water, the longer its sinking travel, the higher the latitude of its point of subduction, and, as the density in the mixed layer increases with latitude, the greater its density.

In consequence, if one wishes to elucidate the density profile, and hence the potential-vorticity input in the interior of the ocean, one must include latitudinal variations and a meridional velocity. Mathematically, the variables now become the mixed-layer depth $h(y,t)$, the meridional velocity $v(y,z,t)$, and the vertical velocity $w(y,z,t)$, where y is the poleward coordinate. In the open ocean where Sverdrup dynamics are a fair approximation, the vorticity balance requires $\beta v = f\partial w/\partial z$ (f is the Coriolis parameter and β its meridional gradient). The trajectory of a parcel in the geostrophic interior must now be constructed from the following equations:

$$\frac{dy}{dt} = v(y,z,t), \; \frac{dz}{dt} = Ek - v(y,-h,t)\frac{\partial h}{\partial y} - \frac{\beta}{f}\int_z^{-h} v(y,z',t)dz', \tag{10}$$

with the following initial conditions $y = y_0$, $z = -h(y_0,t_0)$ at $t = t_0$. [In (10), the vertical velocity at the base of the mixed layer was obtained from (2) and (4).] As one can immediately notice, the problem has become quite complex.

Again, we now choose a simple example to illustrate a few basic concepts. Let us take a mixed layer whose depth varies in time but not in space

$$h(t) = 50 + 30\cos(2\pi t), \tag{11}$$

and meridional–velocity and Ekman–pumping distributions that vary with latitude but not time

$$v(y) = y \quad , \quad Ek(y) = 4.2\ y. \tag{12}$$

Here, time is measured in years (t = 0 is March 1), y in degrees of latitude (y = 0 is 45°N), h in meters, v in degrees of latitude per year, and w in meters per year. [In this unit system, f = 3200 + 67y per year.] This example is thought to mimic the northern part of the Subtropical Gyre (from 30°N to 45°N, $-15<y<0$) where the Ekman pumping is downward and the meridional velocity southward, and the southern part of the Subpolar Gyre (from 45°N to 60°N, $0<y<15$) where the Ekman pumping is upward and the meridional velocity northward.

From expressions (4), (11) and (12), one can immediately determine the rate of subduction:

$$Su = \frac{\partial h}{\partial t} + Ek = -60\pi sin(2\pi t) + 4.2y. \tag{13}$$

Units are meters per year. In a latitude–time plot (Figure 4), the curve Su = 0 is the truncated sine curve y = 45 sin (2πt). Subduction (Su<0) occurs mostly, but not exclusively, during mixed–layer retreat ($0<t<0.5$), even in the northern part of the model where the Ekman pumping is directed upward. This is because the mixed layer retreats faster than the upwelling rate (see Figure 3c).

Interior trajectories are obtained from

$$y = y_0 \exp(t - t_0)$$

where y_0 and t_0 are the latitude and time of subduction, and from the numerical integration of

$$\frac{dz}{dt} = \left[4.2 + \frac{67(z + 50 + 30cos(2\pi t))}{3200 + 67y}\right] y, \tag{14a}$$

$$z(t_0) = -50 - 30cos(2\pi t_0). \tag{14b}$$

The complete set of all trajectories is doubly infinite (at a given time, both y and z can be varied), and cannot be plotted without clutter. Only particular trajectories will thus be constructed, those which delimit bodies of deep, temporarily subducted, and effectively subducted waters. We know from the previous model that these particular trajectories are those which graze the mixed–layer base.

In the southern half of the domain (from 30°N to 45°N), a marginal trajectory, that of a last parcel barely escaping recapture by the deepening mixed layer, is one that is tangent to the mixed–layer base at the time when the latter is near its deepest point (see previous model). Marginal trajectories thus pass at latitudes and times where subduction vanishes (Su=0) passing from positive (end of recapture) to negative (start of subduction). With that branch of the Su=0 curve of Figure 4 as a set of 'initial' conditions, trajectories were integrated backward in time until their previous intersection with the mixed-layer base. These trajectories are plotted on the latitude-time graph of Figure 5, where the intersections with the mixed-layer base are marked with stars. The set of these marginal trajectories defines a curved surface in the three-dimensional (y,z,t) space, lying at all times beneath the mixed-layer base. Sandwiched between that surface and the mixed-layer base are temporarily subducted waters, while under this surface flow the effectively subducted waters.

In the northern half of the domain (from 45°N to 60°N), a marginal trajectory, that of a last parcel rising from the deep and barely missing capture by the mixed layer that year, is one that is tangent to the mixed-layer base at the time when the latter is starting to retreat (see Figure 3c). Marginal trajectories of this kind thus pass at latitudes and times where subduction vanishes (Su=0) passing from positive (end of recapture) to negative (start of subduction). With that branch of Su=0 curve of

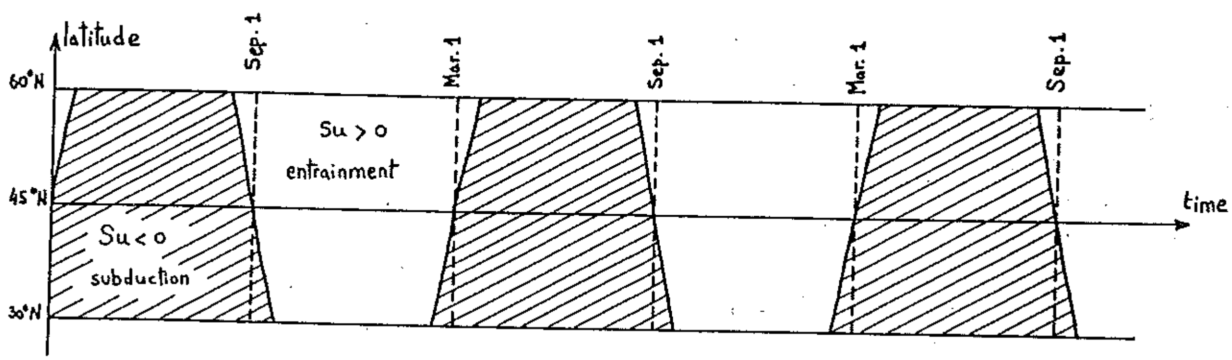

Figure 4. Latitude-time graph showing locations and times when subduction and entrainment occur.

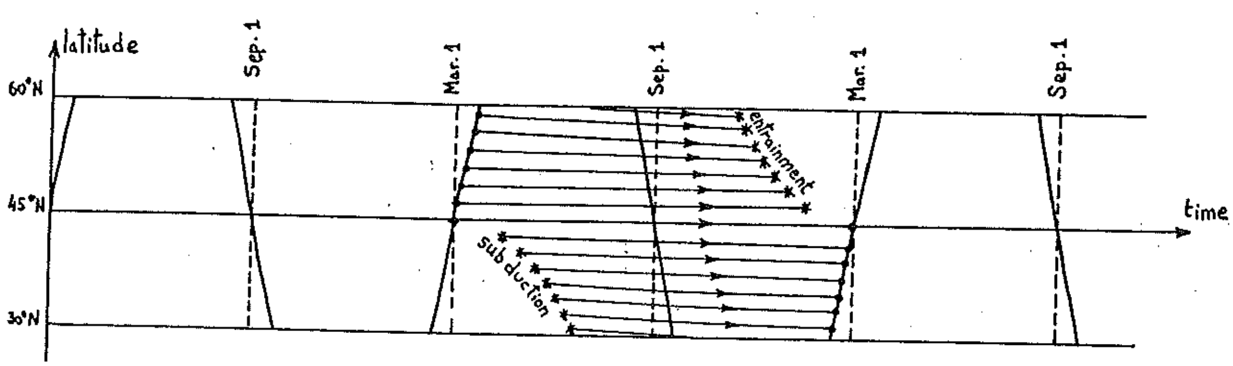

Figure 5. Latitude-time projection of trajectories that graze at one time the base of the mixed layer. Grazing points are marked with heavy dots, while previous or future intersections with the mixed-layer base are marked with stars.

Figure 4 as a set of initial conditions, trajectories were integrated forward in time until their intersection with the mixed-layer base the following winter. These trajectories are also plotted on the latitude-time graph of Figure 5, and, again, the stars mark the intersections with the mixed-layer base. The new curved surface in the three-dimensional (y,z,t) space now separates the lower waters rising from the deep, which will be captured the following winter, from the upper, temporarily subducted waters.

Figure 6 recapitulates the action taking place at the base of the mixed layer. Altogether, four processes occur: permanent subduction, temporary subduction, entrainment of previously subducted waters, and entrainment of deep, upwelling waters. Only up to three process can take place at a given location.
The above analysis sheds light on the data required for separate models of the mixed layer and interior circulation. A mixed-layer model requires data on the deep waters and their rate of upwelling; an interior-circulation model requires information on the variability of the mixed-layer base and on the characteristics of the effectively subducted waters.

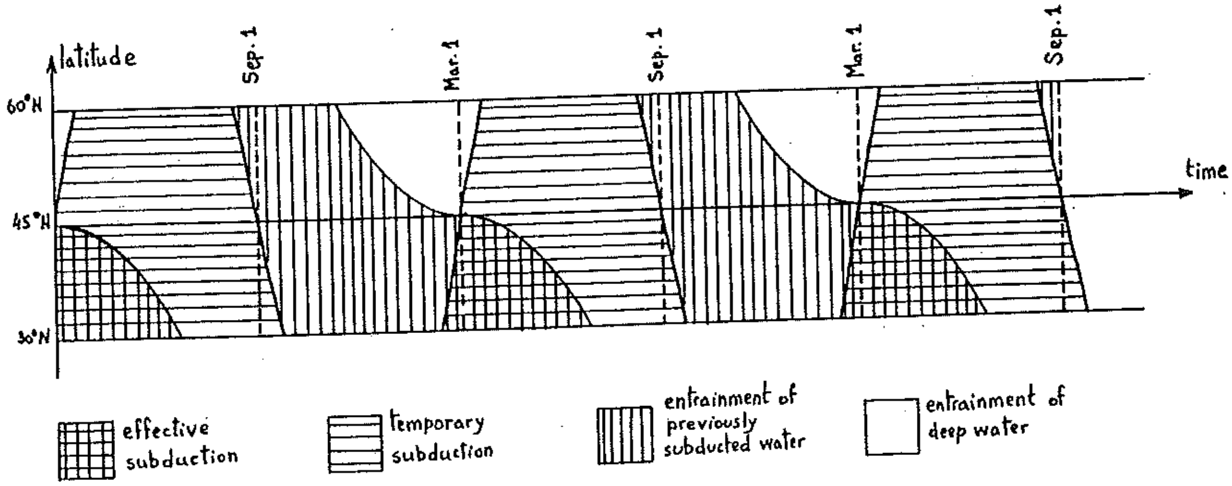

Figure 6. Recapitulation of the various subduction and entrainment regimes in the latitude-depth-time model.

The above analysis also brings up a critical parameter. If the meridional velocity is on the order of V, the meridional excursion of a parcel in a year is on the order of VT (T=one year). But, over that meridional span, the Ekman pumping changes by an amount on the order of VTdEk/dy. If that change is about as large as the Ekman pumping itself, knowing where the parcel originates really matters. Therefore, a critical dimensionless number is the ratio

$$\frac{VT}{Ek}\frac{dEk}{dy}, \tag{15}$$

where V is the magnitude of the interior meridional velocity, Ek is the magnitude of the Ekman pumping, and T is one year. If this ratio is on the order one, latitudinal variations are crucial. For the large-scale circulation numbers quoted earlier, it is found that the ratio is about one-third. Hence, the scenario effective subduction/temporary subduction/entrainment is sensitive to meridional gradients. In other words, determining what does the mixed layer entrain and what does the interior circulation receive requires proper treatment of meridional variations.

LONGITUDE-LATITUDE-DEPTH-TIME MODEL

In the actual ocean, zonal variations can also be important. With the addition of a fourth variable, graphical exploration becomes useless, but no new process is anticipated, the situation being only more intricate in space and time. A new critical ratio, however, now appears. It is

$$\frac{UT}{Ek}\frac{dEk}{dx}, \tag{16}$$

modelled after the previous one, where U is the magnitude of the zonal interior flow and x is the zonal coordinate. If this number is on the order of unity, zonal variations ought to be retained. Such can be the case in the northeast Atlantic Ocean, where the zero-Ekman-pumping line tilts appreciably from southwest to northeast.

DENSITY AND POTENTIAL-VORTICITY

A major question in the theory of large-scale circulation and of the main thermocline is the determination of the density and potential vorticity which are injected into the geostrophic interior. This question is now considered.

At a given location in the interior, the density is subject to fluctuations due to the intermittency of subduction and the seasonal variability of the subducted waters. But, if a requirement is met between interior flow, mixed-layer depth and mixed-layer density, the interior density will not vary in time. Our objective is now to establish this requirement and to determine how close actual ocean conditions come to satisfying it. For that purpose, let us assume that the interior flow (u,v,w) is steady and spatially uniform and that the mixed-layer characteristics are given in terms of its depth, h(x,y,t), and density, ρ(x,y,t).

An interior trajectory obeys $x = x_0+u(t-t_0)$, $y = y_0+v(t-t_0)$, $z = -h(x_0,y_0,t_0) + w(t-t_0)$ where (x_0,y_0) is the location where the parcel was subducted, and t_0 is the time when subduction took place. The density at the point (x,y,z) in the interior at any time, t, is equal to the mixed-layer density at subduction time, $\rho(x_0,y_0,t_0)$, because the interior flow conserves density. For this density to be constant in time, one must have $\partial\rho(x_0,y_0,t_0)/\partial t = 0$ or

$$\frac{\partial \rho}{\partial x}\frac{\partial x_0}{\partial t} + \frac{\partial \rho}{\partial y}\frac{\partial y_0}{\partial t} + \frac{\partial \rho}{\partial t}\frac{\partial t_0}{\partial t} = 0. \tag{17}$$

As time is varied at the fixed location (x,y,z), the coordinates of the trajectory satisfy

$$0 = \frac{\partial x_0}{\partial t} + u - u\frac{\partial t_0}{\partial t}, \quad 0 = \frac{\partial y_0}{\partial t} + v - v\frac{\partial t_0}{\partial t}, \tag{18a,b}$$

$$0 = -\frac{\partial h}{\partial x}\frac{\partial x_0}{\partial t} - \frac{\partial h}{\partial y}\frac{\partial y_0}{\partial t} - \frac{\partial h}{\partial t}\frac{\partial t_0}{\partial t} + w - w\frac{\partial t_0}{\partial t}. \tag{18c}$$

Elimination of $\partial x_0/\partial t$ and $\partial y_0/\partial t$ with (18a) and (18b) transforms (17) and (18c) into

$$\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y}\right)\frac{\partial t_0}{\partial t} = u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y},$$

$$\left(\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y} + w\right)\frac{\partial t_0}{\partial t} = u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y} + w.$$

These last two equations for $\partial t_0/\partial t$ are compatible only if

$$\left(u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y} + w\right)\frac{\partial \rho}{\partial t} = \left(u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y}\right)\frac{\partial h}{\partial t}. \tag{19}$$

This is the requirement between the interior flow (u,v,w) and the mixed-layer characteristics (h,ρ). If we take the liberty to approximate the ocean conditions to understand better what (19) holds, we can neglect the zonal variations in front of the meridional variations and note that the density varies meridionally substantially more than the mixed-layer depth. Requirement (19) then becomes a balance between its two prominent terms:

$$w\frac{\partial \rho}{\partial t} \simeq v\frac{\partial \rho}{\partial y}\frac{\partial h}{\partial t}. \tag{20}$$

In a northern-hemisphere Subtropical Gyre where the interior flow is southward and downward (v and w negative), effective subduction takes place during mixed-layer retreat ($\partial h/\partial t$ negative), which is also the time of year when the mixed-layer temperature rises ($\partial \rho/\partial t$ negative). Balance (20) requires $\partial \rho/\partial y$ positive as a necessary condition for a constant density in the interior. This is the case in the ocean, where density increases northward. Physically, the situation is as follows: the waters passing at a given location and depth in the interior, have been subducted at different times; the more recent waters have been deposited somewhat north of that location when the mixed layer was deep and winter-cold; the older waters must come from further north where the mixed layer is colder but have been subducted when the mixed layer was not as deep. If the signs of the quantities in (20) favor a balance, it still remains to check the magnitudes of the various terms. For $w \simeq 10^{-6}$m/s

(30m/year), $v \simeq 10^{-2}$m/s, $\partial\rho/\partial t$ = 0.6 sigma units in half a year, $\partial\rho/\partial y$ = 0.6 sigma units in 10° latitude, and $\partial h/\partial t$ = 100 meters in half a year, the ratio of the left-hand side of (20) over the right-hand side is approximately unity. Therefore, it turns out that the Subtropical Gyre in the ocean is naturally satisfying the requirement for a steady interior.

The next question is how to establish the potential-vorticity input function for the interior flow in terms of the mixed-layer variable. In a large-scale flow, the potential vorticity is simply PV = $-f\partial\rho_I/\partial z$, where ρ_I is the interior density. To evaluate the vertical gradient of density in the interior, let us first take a first parcel subducted at location (x,y) and time t; it was released at depth z = $-h(x,y,t)$ and with a density $\rho(x,y,t)$. During the interval of time Δt immediately following subduction, it sinks to depth $z+\Delta z = -h(x,y,t) + w\Delta t$ while it migrates laterally to the location ($x+\Delta x = x+u\Delta t$, $y+\Delta y = y+v\Delta\tau$). At that location and at that time, the mixed layer has depth $h(x+\Delta x,y+\Delta y,t+\Delta t)$ and released a new parcel of density $\rho(x+\Delta x,y+\Delta y,t+\Delta t)$, just above our previous parcel. If the time interval, Δt, is infinitesimal, the vertical density gradient can be evaluated as follows

$$\frac{\partial\rho_I}{\partial z} = \frac{\rho(x+\Delta x, y+\Delta y, t+\Delta t) - \rho(x,y,t)}{-h(x+\Delta x, y+\Delta y, t+\Delta t) + h(x,y,t) - w\Delta t}$$

$$= -\frac{\rho_t\Delta t + \rho_x\Delta x + \rho_y\Delta y}{h_t\Delta t + h_x\Delta x + h_y\Delta y + w\Delta t} \qquad (21)$$

and the potential vorticity is found to be

$$PV = f\frac{\rho_t + u\rho_x + v\rho_y}{h_t + uh_x + vh_y + w} = \frac{f}{Su}(\rho_t + u\rho_x + v\rho_y). \qquad (22)$$

The reader is reminded that, in expression (22), h and ρ are the mixed-layer variables while u,v and w are the velocity components in the interior. Finally, if requirement (19) for a steady interior is met, then expression (22) simplifies to

$$PV = f\frac{\rho_t}{h_t}. \qquad (23)$$

CONCLUSIONS

Although the above considerations have not provided an exhaustive treatment of subduction, several key questions were clarified. First, the common belief that the rate of subduction equals the rate of Ekman pumping was proven not to hold true, even after a suitable average over the mixed-layer cycle. The differences between rate of mixed-layer deepening, Ekman pumping, fluid vertical velocity, and rate of subduction have been clarified in the text.

Second, the intermittency of seasonal subduction was investigated with a hierarchy of models. It was found that subduction can be effective (permanent) or only temporary, and that effective subduction can take place over several months, as the mixed layer retreats. Critical dimensionless numbers were identified: (i) the ratio of interior vertical velocity over the maximum deepening rate, which controls the efficiency of subduction and hence the characteristics of the subducted waters, (ii) the ratio of meridional displacement during one year over the meridional distance over which the Ekman pumping changes appreciably, and (iii) the analogous ratio in the zonal direction. These last two numbers indicate the importance of lateral variations in subduction.

Third, it was found that the Subtropical-Gyre regions of the oceans, where subduction controls the permanent thermocline, naturally lead to a constant density field thanks to a competition by which seasonal intermittency of subduction negates variability in the subducted waters. Finally, a formula was constructed to determine the potential-vorticity injection into the interior in terms of the interior velocity flow at the base of the mixed layer and the mixed-layer depth and density.

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