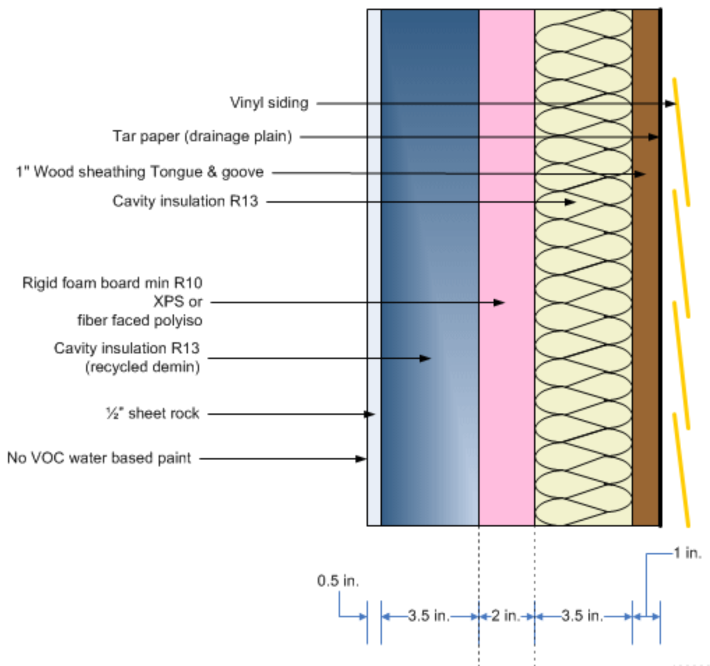
ENGS 44 **Winter 2012**
SUSTAINABLE DESIGN

HOMEWORK #1 SOLUTIONS

1. (10 points) What is the *R*-value of the wall section depicted below? Do you think it is on the low side, average, or on the high side?



The R-value of this wall is simply the sum of the R-values of the individual layers making up this wall:

R = 0.68 for still air layer on inside
+ 0.00 for water-based paint (too thin to count)
+ 0.64 for half-inch sheet rock
+ 13.0 for cavity insulation (recycled denim)
+ 10.0 for pink rigid foam board
+ 13.0 for cavity insulation
+ (1 inch)(0.95/inch) for 1-inch wood sheathing tongue and groove
+ 0.00 for tar paper (too thin to count)
+ 0.61 for vinyl siding
(http://www.coloradoenergy.org/procorner/stuff/r-values.htm)
+ 0.17 for outdoor air layer in presence of 15 mph wind
= 39.05
say **39.**

This value can be considered to be on the **HIGH SIDE**.

2. (10 points) Next to my house in Lebanon NH is a small one-room rectangular shed that I wish to convert into a heated studio. For this, it needs to be better insulated. The floor is 16 ft long by 12 ft wide, walls are 7 ft tall, the roof surface is 263 ft^2, and there are three windows occupying a total area of 72 ft^2. One of the two short sides is common with my house, and the door is along that side.

If I do not want to spend more than 10 x 10^6 BTUs per year for heating this shed converted into a studio, what should be the *R*-values of the walls, windows and roof? Neglect heat losses through the floor and by air infiltration. Select the various *R*-values so that walls, windows and roof contribute, approximately, 20%, 60% and 20% to the heat loss. Should I use double-pane or triple-pane windows?

The wall area, minus windows, is:

A_{walls} = sum of lengths of 3 sides times height minus window area
= (16 + 12 + 16) x 7 - 72 = 308 - 72 = 236 ft^2.

Other areas are given:

A_{windows} = 72 ft^2

A_{roof} = 263 ft^2

The total heat load (HL) of the shed must not exceed

$$HL < \frac{\text{energy ceiling}}{\text{degree} \cdot \text{days}} = \frac{10 \times 10^6 \ BTUs/year}{7{,}694 \ ^0F \cdot days \cdot year} = 1{,}300 \ BTUs/^0F \cdot day$$

● 20% of heat loss through walls

→ (0.20)(1,300) = 259.94 BTUs/^{o}F.day = 10.83 BTUs/^{o}F.hour

$$R_{wall} = \frac{236 \ ft^2}{10.83 \ BTUs/^0F \cdot hour} = 21.79$$ → Take R_{wall} = 22

corresponding U-value is $U_{\text{wall}} = 1/R_{\text{wall}} = 0.0459$ BTUs/ft^2.hr.^{o}F

- 60% of heat loss through windows
  → (0.60)(1,300) = 779.83 BTUs/^{o}F.day = 32.49 BTUs/^{o}F.hour

$$R_{windows} = \frac{72 \ ft^2}{32.49 \ BTUs/^0F \cdot hour} = 2.22$$

corresponding U-value is $U_{\text{windows}} = 1/R_{\text{windows}} = 0.451$ BTUs/ft^2.hr.^{o}F

- 20% of heat loss through roof
  → (0.20)(1,300) = 259.94 BTUs/^{o}F.day = 10.83 BTUs/^{o}F.hour

$$R_{roof} = \frac{263 \ ft^2}{10.83 \ BTUs/^0F \cdot hour} = 24.28 \ \rightarrow \text{ Take } R_{\text{roof}} = 25$$

corresponding U-value is $U_{\text{roof}} = 1/R_{\text{roof}} = 0.041$ BTUs/ft^2.hr.^{o}F

Selection of windows from among available types:

double-pane windows have $R = 1.92$ ← closer to the 2.22 needed
triple-pane windows have $R = 5.88$ ← in excess of the 2.22 needed

So, go with double-pane windows.

Now, re-do calculations since R_{windows} is a bit different.

Heat loss through windows = U_{windows} x A_{windows} = (1/1.92)(72)
= 37.50 BTUs/^{o}F.hr = 900.0 BTUs/^{o}F.day

This is a bit more than before (900 > 780). Thus, revise heat loss of walls and roof.

Walls and roof may not leak more than allocated:

$HL_{\text{wall}} + HL_{\text{roof}} = HL_{\text{total}} - HL_{\text{windows}}$
= 1,300 - 900.0 = 400.0 BTUs/^{o}F.day

Splitting this subtotal into two equal contributions (refer to initial percentages):

$HL_{\text{wall}} = HL_{\text{roof}} = ½$ of 400.0 BTUs/^{o}F.day
= 200.0 BTUs/^{o}F.day = 8.333 BTUs/^{o}F.hr

From this follows the required R-values for the walls and roof:

$$R_{wall} = \frac{236\ ft^2}{8.333\ BTUs/{}^0F \cdot hour} = 28.32 \quad \rightarrow \text{ Take } R_{\text{wall}} = 28 \text{ or } 29$$

$$R_{roof} = \frac{263\ ft^2}{8.333\ BTUs/{}^0F \cdot hour} = 31.56 \quad \rightarrow \text{ Take } R_{\text{roof}} = 32$$

3. (10 points) Consider the salt-box house in Lebanon taken as the example during Prof. Cushman-Roisin's lecture and suppose that its owner decides not to live in it during the four coldest months of the year (December, January, February and March). To avoid the freezing of pipes, however, he intends to set the thermostat at 40°F. What are the revised degree-days for the house and how much money will he save at the rate of 9 cents per kWh? (Conversion factor: 10^6 BTUs = 293.1 kWh.)

When the indoor temperature drops from the canonical value of 65ºF to the new value of 40ºF (no benefit from body heat & appliances):

$$\Delta T = 65°\text{F} - 40°\text{F} = 25°\text{F}$$

Therefore, there will be 25ºF less during the: 31 days of December
31 days of January
28 days of February
+ 31 days of March
121 days total

The reduction in degree days is $(25°\text{F}) \cdot (121\,\text{days}) = 3{,}205\,\text{degree} \cdot \text{days}$

The *HL* value of the salt-box house (with infiltration!) is 10,880 BTUs/day.°F.

The energy savings is then $\left(10{,}880 \dfrac{\text{BTUs}}{\text{day}°\text{F}}\right)\left(3{,}025°\text{F} \cdot \text{day}\right) = 32{,}912{,}000\,\text{BTUs}.$

Conversion factor is 1 kWh = 3412 BTUs → savings of 9,646 kWh
(or 10^6 BTUs = 293.1 kWh).

@ $0.14/kWh, the monetary savings are (9,646 kWh)($0.14/kWh) = **$1,350.44.**

4. (10 points) Consider again the salt-box house in Lebanon taken as an example during Prof. Cushman-Roisin's lecture and suppose now that its owner approaches you about replacing eight of the existing 2½ft x 4ft windows with higher *R*-value windows in order to save on heating fuel. The owner wants to have payback on the investment in five years or less.

The following windows are available on the market, with prices inclusive of installation:

No.	**Type**	***U*-value**	***R*-value**	**cost**
1	Single hung, low-E2	0.42	2.38	$245
2	Wood clad, low-E	0.32	3.125	$336
3	Low-E Solarban 70, argon	0.28	3.57	$419
4	Elite triple pane	0.23	4.35	$510
5	2 TiAC40 low-E	0.17	5.88	$705

Heating fuel delivers 139,200 BTUs per gallon at a cost of $3.13/gallon. The owner wants to maximize return on the investment and is not interested in minimizing carbon footprint. Do you advise the owner to go ahead with the retrofit and, if so, which windows do you recommend?

Do conclusions remain the same if fuel cost escalates to $7/gallon?

To begin, let us express the Heat Loss *HL* in terms of the floating *R* value of the windows. Since the heat loss is changed for only 8 windows of 2.5ft x 4ft = 80 ft², it is unchanged for the remaining window area (271 ft² – 80 t² = 191 ft²), walls, roof and infiltration. Thus, sequentially for walls, untouched windows, new windows, roof and infiltration, we have:

$$HL = \left(\frac{1{,}898\ ft^2}{21.37} + \frac{191\ ft^2}{1.92} + \frac{80\ ft^2}{R} + \frac{1{,}520\ ft^2}{31.97} \right) \times 24 \frac{hrs}{day} + 4{,}220 \frac{BTUs}{day \cdot {}^0\ F}$$
$$= \left(9{,}880 + \frac{1{,}920}{R} \right) \frac{BTUs}{day \cdot {}^0\ F}.$$

The energy load of the house is this *HL* value times the number of degree.days:

$$Heat\ Load = HL \times DD = \left(9{,}880 + \frac{1{,}920}{R} \right) \times 7{,}694 = \left(76{,}017{,}921 + \frac{14{,}772{,}480}{R} \right) BTUs.$$

The energy saving is the past energy consumption (preceding expression with *R* = 1.92) minus the new energy consumption (preceding expression with floating *R* value):

$$Energy\ Saving = \left(\frac{14{,}772{,}480}{1.92} - \frac{14{,}772{,}480}{R} \right) = \left(7{,}694{,}000 - \frac{14{,}772{,}480}{R} \right) BTUs.$$

The cost of fuel on an energy basis is known to be:

$$Fuel\ Cost = \frac{\$3.13 / gallon}{139{,}200\, BTUs / gallon} = \$2.2486 \times 10^{-5}\ \ per\ BTU.$$

Thus, the cost saving of the retrofit is:

$$Cost\ saving = \left(7{,}694{,}000 - \frac{14{,}772{,}480}{R} \right) BTUs \times \frac{\$2.2486 \times 10^{-5}}{1\, BTU} = \$173.00 - \$\frac{332.17}{R}.$$

The cost of windows is simply 8 times the cost of 1 window.

The cost comparison can now be established:

Window No.	*R*-value	Cost saving	Savings cumulated over 5 yrs	Unit cost	Cost of retrofit	Net cost of retrofit over 5 years
1	2.38	$33.489	$167.44	$245	$1,960	$1,793
2	3.125	$66.706	$333.53	$336	$2,688	$2,354
3	3.57	$79.992	$399.96	$419	$3,352	$2,952
4	4.35	$96.601	$483.00	$510	$4,080	$3,597
5	5.88	$116.531	$582.66	$705	$5,640	$5,057

As it turns out, no matter which window type is selected, the cost of retrofit always exceeds the savings in reduced fuel consumption over 5 years, and there is no economic incentive to do the proposed upgrade. The net loss actually increases with the *R*-value of the window. Thus, the better the window, the higher the net cost.

If fuel cost escalates to $7, the fuel savings increase in the ratio of 3.13 over 7.00 but the conclusion remains unchanged: No matter which window type is selected, the cost of retrofit always exceeds the savings.