

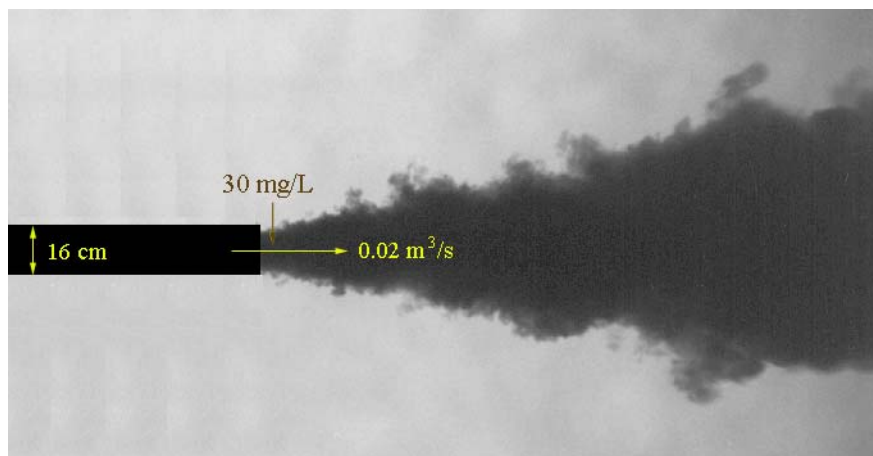
HOMEWORK #4

Assigned: Friday 27 January 2012

Due: 11:15 a.m., Friday 3 February 2012

1. (10 points) An underwater pipe of 16 cm inner diameter discharges $0.02 \text{ m}^3/\text{s}$ of wastewater containing 30 mg/L of nitrogen (in the form of nitrates) into an otherwise pristine lake. What are the nitrogen concentrations 10 m, 20 m and 50 m away from the pipe in the direction of the jet flow? At which distance has the concentration fallen to 0.5 mg/L?

A nozzle or diffuser can be fitted onto the end of the pipe to reduce or enlarge the exit diameter. What should that diameter be to ensure that the concentration of nitrogen falls to 0.5 mg/L at a distance of 15 m downstream of the pipe's end?



2. (10 points) A jet of clean water enters a quiescent pond where the water is contaminated. What is the concentration distribution $c(x, r)$ as function of downstream distance x and radial distance r if the jet velocity is U , the pipe inner diameter d , and the ambient concentration in the quiescent pond is c_0 ? What are the dimensions (length and maximum width) of the volume of water inside which the concentration c lies below $c_0/2$?

3. (10 points) In summer, the discharge of the Po River into the northern Adriatic Sea creates regionally a 2.5m-thick surface layer of fresher and warmer water overlying saltier and colder bottom water. The total water depth in the region is 28 m, and the respective water densities are 1024.3 and 1025.7 kg/m³. During a storm, winds set up a sea current of 0.50 m/s that extends vertically over the depth of the fresher-warmer layer, while there remains no appreciable current in the bottom layer. Mixing ensues from the velocity difference across layers. Is there enough shear in velocity to create mixing all the way down to the bottom?

If the phosphorus concentration in the surface layer due to the Po River was 3.0 mg/L and the bottom concentration was 1.2 mg/L before the storm, what is the surface phosphorus concentration after the storm?

4. (20 points) The following five questions concern an upstream stretch of the White River in Vermont, which has an averaged slope of 5×10^{-5} and an averaged width of 6 m. The roughness of its bed corresponds to a Manning coefficient $n = 0.040$. In summer, the averaged depth and temperature are 1.2 m and 12°C, respectively.

- a. (5 points) Estimate the averaged velocity in the river and the diffusion coefficients in all three directions (vertical, transverse and longitudinal). Determine also the reaeration coefficient (in per-day units).
- b. (5 points) Treated sewage (with a decay constant of 0.85 /day at 20°C) is continuously discharged from a pipe at mid-depth along the right bank. Over which downstream distance does it become well mixed in both vertical and transverse directions? Is decay significant during this stage?
- c. (5 points) Show that if longitudinal diffusion is neglected, the distance over which the pollutant remains in excess of 25% of the value at the point where cross-sectional mixing has been achieved is proportional to the river velocity and inversely proportional to the sewage decay constant. What is that distance in the present case?
- d. (5 points) Using the preceding distance as the relevant downstream length scale, can you show that longitudinal diffusion is indeed negligible?