

ENGS-43, Winter 2012
ENVIRONMENTAL TRANSPORT & FATE

HOMEWORK #1 – SOLUTIONS

1. (10 points) Consider the so-called dead-zone off the Mississippi River Delta in the Gulf of Mexico (<http://serc.carleton.edu/microbelife/topics/deadzone/>). The chief problem is excessive nitrogen and phosphorus runoff from agriculture in the watershed of the Mississippi River. From the facts of the situation, cite two possible types of remedial actions for each of the three possible approaches (management at the source, along the path and at the receptor level), for a total of six possible remedial actions. A couple of lines on each action constitutes a sufficient answer.

Source management:

The given website suggests the following four remedial actions at the source:

- Using fewer fertilizers and adjusting the timing of fertilizer applications to limit runoff of excess nutrients from farmland.
- Control of animal wastes so that they are not allowed to enter into waterways.
- Monitoring of septic systems and sewage treatment facilities to reduce discharge of nutrients to surface water and groundwater.
- Careful industrial practices such as limiting the discharge of nutrients, organic matter, and chemicals from manufacturing facilities.

Pathway management:

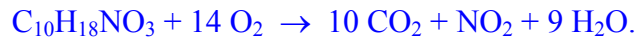
- A relatively practical remediation along the pathway is to restore wetlands along the Mississippi River to slow down water and allow nutrient-rich sediments to settle and fertilize land rather than the sea further downstream.
- An alternative is to construct several filtering stations along the course of Mississippi River to remove nutrients along the way. (Such methods exist and are used in wastewater treatment plant that perform so-called “tertiary treatment”.)

Receptor management:

This, of course, is the most brutal and least desirable approach, but possibilities nonetheless exist. Some are:

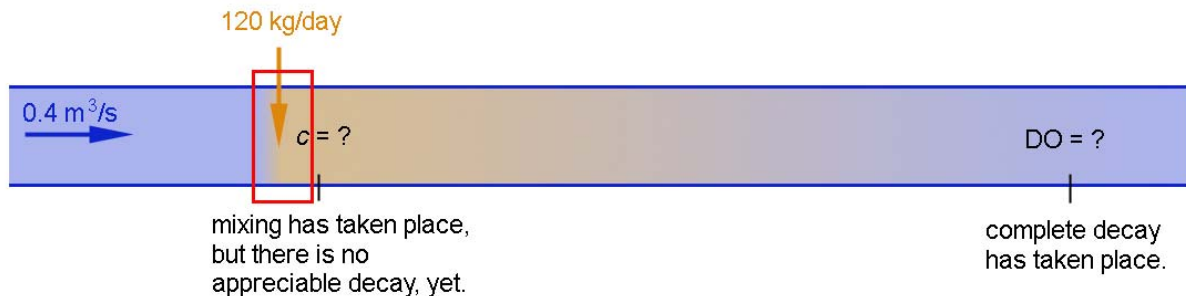
- Patrol the dead-zone area with a flotilla of boats that inject oxygen in the water to prevent hypoxia.
- Inject in the water at the mouth of the Mississippi and wherever an algal bloom arises some herbicide-type chemicals that kills the unwanted algae.

2. (10 points) A certain pre-treated sewage consists of organic material with average chemical formula $C_{10}H_{18}NO_3$, which slowly decays in several biological and chemical steps that can be summed as the net chemical reaction:



If 120 kg/day of this type of sewage is continuously discharged in a small stream carrying $0.4 \text{ m}^3/\text{s}$ of water with 9.3 mg/L of dissolved oxygen upstream, determine

- (a) (5 points) the pollutant concentration in the stream shortly downstream of the discharge (assume nearly immediate vertical and transverse mixing across the stream but no appreciable decay, yet), and
- (b) (5 points) the ultimate concentration of dissolved oxygen (in mg/L) in the water further downstream after total degradation of the pollutant?



(a) Concentration immediately after mixing of the discharge:

For this, we write the budget for a small portion of the stream from immediately upstream of the discharge to just downstream of where mixing has been completed (see red box on figure). In steady state (with $dc/dt = 0$), it is

$$0 = Q_{in}c_{in} - Q_{out}c_{out} + S - KVC$$

The first term drops out because $c_{in} = 0$ (clean water upstream of discharge). In the second term, we replace Q_{out} by Q (the stream's volumetric flowrate) and c_{out} by c . Then, by assuming – as the problem statement allows us to do – that there is no decay in this short zone, we drop the last term. The budget reduces to

$$0 = -Qc + S$$

The solution of which is immediate

$$\begin{aligned} c &= \frac{S}{Q} = \frac{(120 \text{ kg/day})}{(0.4 \text{ m}^3/\text{s})(86,400 \text{ s/day})} \\ &= 0.00347 \frac{\text{kg}}{\text{m}^3} = 0.00347 \frac{\text{g}}{\text{L}} = 3.47 \frac{\text{mg}}{\text{L}} \end{aligned}$$

(b) Loss of dissolved oxygen:

For this we need to consider the chemical balance, which tells us that

One “**molecule**” of sewage takes up 14 **molecules** of oxygen.

Go from molecules to moles (= multiply by the Avogadro number), to get

One **mole** of sewage takes up 14 **moles** of oxygen.

Next, convert moles into grams by using molecular weights:

Molecular weight of $C_{10}H_{18}NO_3 = 12 \times 10 + 1 \times 18 + 14 \times 1 + 16 \times 3 = 200$ grams

Molecular weight of $O_2 = 16 \times 2 = 32$ grams.

which means that

200 **grams** of sewage destroy $14 \times 32 = 448$ **grams** of oxygen.

Divide by 200:

1 gram of sewage destroys $448/200 = 2.24$ grams of oxygen.

We can then pass from grams to milligrams and on to milligrams per liter:

1 **mg/L** of sewage destroys 2.24 **mg/L** of oxygen.

Since we have an initial concentration of 3.47 mg/L of sewage, the ultimate destruction of oxygen (after all sewage has decayed) is:

3.47 mg/L of sewage destroys $2.24 \times 3.47 = 7.78$ mg/L of oxygen.

Since the upstream condition is $DO = 9.3$ mg/L, the remaining portion of dissolved oxygen far downstream is

$$\begin{aligned} DO_{\text{downstream}} &= DO_{\text{upstream}} - DO_{\text{removed}} \\ &= 9.30 - 7.78 = \mathbf{1.52 \text{ mg/L}}. \end{aligned}$$

3. (10 points) A 200-m³ swimming pool has been overly chlorinated by accidentally dumping an entire container of chlorine, which amounted to 500 grams. This creates a chlorine concentration well in excess of the 0.20 mg/L that was the objective at the time. With which volumetric rate (in L/min) of freshwater should the pool be flushed to decrease the concentration to the desired level in 36 hours? You may assume that the chlorine does not degrade chemically during those hours.

The initial concentration is $c_o = \text{amount dumped over volume of pool} = 500 \text{ g} / 200 \text{ m}^3 = 2.5 \text{ g/m}^3 = 2.5 \text{ mg/L}$.

In the absence of a persistent source and of decay, the chlorine budget during the flushing period is:

$$V (dc/dt) = Q_{in} c_{in} - Q_{out} c_{out}$$

where $c_{in} = 0$ because one flushes with clean water, $Q_{out} = Q$, the flushing volumetric flowrate, and $c_{out} = c$, the chlorine concentration inside the pool at the moment. It reduces to

$$V (dc/dt) = - Q c$$

and the solution is

$$c(t) = c_o \exp(- Q t / V)$$

where c_o is the initial concentration, determined above. Since we know the concentration c (= 0.20 mg/L) that we want to achieve and by which time t (= 36 hours), the unknown here is the flowrate Q . Solving for it, we get:

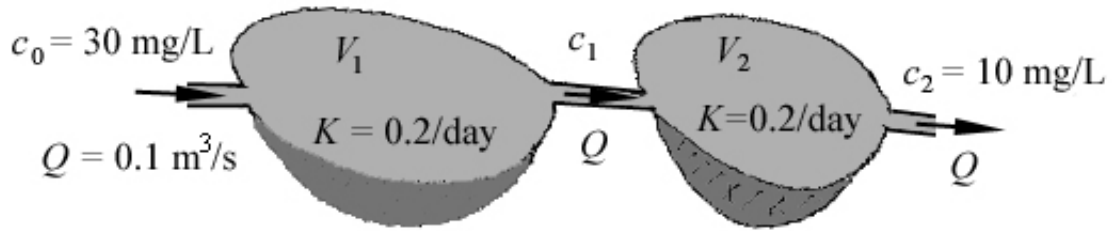
$$Q = (V / t) \ln(c_o / c)$$

Plugging the numbers, we obtain

$$Q = (200 \text{ m}^3 / 36 \text{ h}) \ln(2.5 \text{ mg/L} / 0.20 \text{ mg/L}) = 14.032 \text{ m}^3/\text{h}$$

Changing the units to liters per minute, we get: **$Q = 234 \text{ L/min}$** .

4. (10 points) A two-lagoon system (see sketch below) is being designed to accommodate an input flow of $0.10 \text{ m}^3/\text{s}$ of a biodegradable pollutant with concentration of 30 mg/L . The pollutant decomposition rate in each lagoon will be 0.2 /day . The effluent from the second lagoon must have a pollutant concentration not exceeding 10 mg/L . Assuming complete mixing in each lagoon and deciding on an optimal volume partitioning between the two lagoons (for you to define – be practical), determine the necessary volume of water in each lagoon. Is there a real advantage in using two lagoons instead of one?



Denote by Q ($= 0.10 \text{ m}^3/\text{s}$) the flow rate through the system, by K ($= 0.2 \text{ /day}$) the decay rate, and by V_1 and V_2 the volumes of the first and second lagoons, respectively, which are unknown.

Denote then by c_0 , c_1 and c_2 , the concentrations upstream, in lagoon 1 and in lagoon 2, respectively. What exits lagoon 1 and enters lagoon 2 has concentration c_1 , and what flows out of lagoon 2 has concentration c_2 . The known values are $c_0 = 30 \text{ mg/L}$ and $c_2 = 10 \text{ mg/L}$ (working at the acceptable limit!). The intermediate value c_1 is unknown.

The separate budgets for the two lagoons in steady state are:

$$0 = Qc_0 - Qc_1 - KV_1c_1$$

$$0 = Qc_1 - Qc_2 - KV_2c_2$$

Solving for the exiting concentrations, we obtain:

$$c_1 = \frac{Q}{Q + KV_1} c_0 \quad \text{and} \quad c_2 = \frac{Q}{Q + KV_2} c_1$$

Eliminating the intermediate concentration c_1 , which is unknown and of no importance to the problem, we obtain:

$$c_2 = \frac{Q}{Q + KV_2} \frac{Q}{Q + KV_1} c_0$$

Dividing by c_0 (and noting that c_2 / c_0 must be $= 10 / 30 = 1 / 3$), we have a requirement between the two volumes:

$$\frac{Q}{Q + KV_1} \frac{Q}{Q + KV_2} = \frac{1}{3}$$

Introducing for convenience the dimensionless volumes $v_1 = KV_1 / Q$ and $v_2 = KV_2 / Q$, this becomes:

$$\frac{1}{(1 + v_1)(1 + v_2)} = \frac{1}{3}, \text{ or } (1 + v_1)(1 + v_2) = 3$$

This makes one relationship between two unknowns, namely v_1 and v_2 . To have an idea of what kind of relationship this is, let us vary v_1 and solve for v_2 and track the total volume $v_1 + v_2$:

$v_1 = 0.000$	\rightarrow	$v_2 = 2.000$	and	$v_1 + v_2 = 2.000$
$v_1 = 0.200$	\rightarrow	$v_2 = 1.500$	and	$v_1 + v_2 = 1.700$
$v_1 = 0.400$	\rightarrow	$v_2 = 1.143$	and	$v_1 + v_2 = 1.543$
$v_1 = 0.600$	\rightarrow	$v_2 = 0.875$	and	$v_1 + v_2 = 1.475$
$v_1 = 0.700$	\rightarrow	$v_2 = 0.765$	and	$v_1 + v_2 = 1.465$
$v_1 = 0.800$	\rightarrow	$v_2 = 0.667$	and	$v_1 + v_2 = 1.467$
$v_1 = 1.000$	\rightarrow	$v_2 = 0.500$	and	$v_1 + v_2 = 1.500$
$v_1 = 1.200$	\rightarrow	$v_2 = 0.364$	and	$v_1 + v_2 = 1.564$
$v_1 = 1.400$	\rightarrow	$v_2 = 0.250$	and	$v_1 + v_2 = 1.650$
$v_1 = 1.600$	\rightarrow	$v_2 = 0.154$	and	$v_1 + v_2 = 1.754$
$v_1 = 1.800$	\rightarrow	$v_2 = 0.071$	and	$v_1 + v_2 = 1.871$
$v_1 = 2.000$	\rightarrow	$v_2 = 0.000$	and	$v_1 + v_2 = 2.000$.

We note that, as v_1 increases, v_2 decreases, which is to be expected because a larger first lagoon promotes more decay and leaves less to be accomplished by the second lagoon.

The sum of the two volumes is not constant but passes through a minimum. This minimum is of interest because it corresponds to the least total volume. At this minimum, the least amount of soil or rocks needs to be excavated to create the set of lagoons, the least material must be transported to a new location, and the least damage is done to the environment. There is therefore an economic gain and an environmental advantage in designing the system so that the total volume is minimized.

By symmetry of the equations, it is evident that the minimum is obtained when $v_1 = v_2$, which yields:

$$v_1 = v_2 = \sqrt{3} - 1 = 0.732$$

Returning to the dimensional variables, we obtain the necessary volumes:

$$V_1 = V_2 = 0.732 \frac{Q}{K} = 0.732 \frac{(0.1 \text{ m}^3 / \text{s})(24 \times 60 \times 60 \text{ s} / \text{day})}{(0.2 / \text{day})} = 31,625 \text{ m}^3$$

If we had designed instead a single-lagoon system, we would have $V_2 = 0$ and

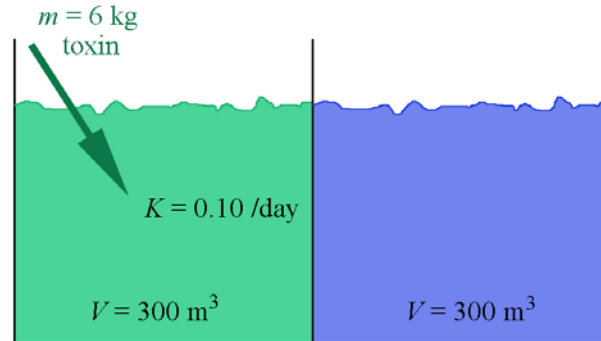
$$v_2 = 0 \rightarrow v_1 = 2 \rightarrow V_1 = 2 \frac{Q}{K} = 2 \frac{(0.1 \text{ m}^3 / \text{s})(86400 \text{ s} / \text{day})}{(0.2 / \text{day})} = 86,400 \text{ m}^3$$

So, for a two-lagoon system, we need to dig two small holes of $31,625 \text{ m}^3$ each, for a total of $63,250 \text{ m}^3$, instead of digging one big hole of $86,400 \text{ m}^3$. This reduces the impact on the land and saves about 27% of the work and expense, and **there is a definite advantage to using a two-lagoon system.**

5. (10 points) You are working for a chemical manufacturing plant, which has two adjacent pools of water (300 m^3 each) for the continuous processing of certain contaminated wastes. One day, 6 kg of a toxic substance is accidentally dropped in one of the pools, and that pool must be temporarily shut down. You know that in the presence of the other contaminants, this toxin decays at a rate of 0.10 per day, and that it will be legal and safe to resume operations when the toxin concentration has fallen below 0.5 mg/L .

You have two options: (1) to wait for the toxin concentration in the contaminated pool to drop to the legal level, while keeping the other pool in operation, and (2) to shutdown the other pool, remove the partition separating the two pools thereby diluting the contaminant, and wait until the concentration of the mixture has dropped to the legal level. The shut-down cost is $\$125$ per pool per day. Which option is more economical?

The situation looks like



1st option: Shut down the contaminated pool and keep the clean pool in operation

Using the contaminated pool as the control volume, the budget is

$$V \frac{dc}{dt} = -K V c$$

with no import and export (because the pool is isolated) and without a source term (because, although there was a momentary release, it does not continue over time).

The solution to this equation is:

$$c(t) = c_0 e^{-Kt}$$

where c_0 is the initial concentration. This initial concentration is simply the mass of toxin that was accidentally introduced divided by the volume of the pool:

$$c_0 = \frac{6 \text{ kg}}{300 \text{ m}^3} = \frac{6 \times 10^6 \text{ mg}}{300 \times 10^3 \text{ L}} = 20 \text{ mg / L}$$

The time t until the concentration has dropped to the acceptable value of 0.5 mg/L is such that

$$(0.5 \text{ mg / L}) = (20 \text{ mg / L}) e^{-Kt} \rightarrow e^{-Kt} = \frac{0.5}{20} = 0.025$$

Taking the natural logarithm and solving for t , we get

$$t = \frac{-1}{K} \ln(0.025) = \frac{+3.689}{(0.10 / \text{day})} = 36.89 \text{ days} \approx 37 \text{ days}$$

At the rate of \$125 per day, the cost of this option is $37 \times \$125 = \mathbf{\$4,625}$.

2nd option: Shut down both pools and remove the partition

This becomes the case of a single pool but with volume equal to twice the volume of the pool of the 1st option: $V = 2 \times 300 \text{ m}^3 = 600 \text{ m}^3$. The initial concentration is then half as strong:

$$c_0 = \frac{6 \text{ kg}}{600 \text{ m}^3} = \frac{6 \times 10^6 \text{ mg}}{600 \times 10^3 \text{ L}} = 10 \text{ mg / L}$$

The new time t that leads to the acceptable value of 0.5 mg/L is

$$(0.5 \text{ mg / L}) = (10 \text{ mg / L}) e^{-Kt} \rightarrow e^{-Kt} = \frac{0.5}{10} = 0.05$$

$$\rightarrow t = \frac{-\ln(0.05)}{(0.10 / \text{day})} = 29.95 \text{ days}$$

In other words, very close to 30 days. The cost of shutting both pools down for this length of time is $(2 \text{ pools}) \times (\$125/\text{pool per day}) \times (30 \text{ days}) = \mathbf{\$7,500}$.

Comparison and decision:

As expected the time required for self-cleaning is shorter when both pools are shut down and the partition removed, but the overall cost is higher. It is thus more economical to shut down only the contaminated pool and **keep the uncontaminated pool in operation**.