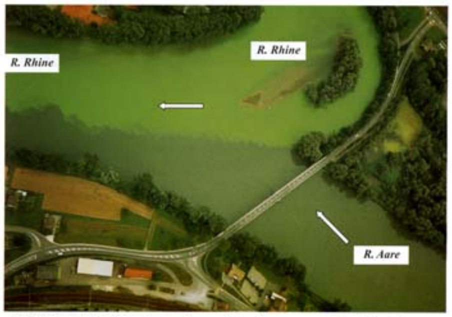
Transport Phenomena

(Mihelcic & Zimmerman, Section 4.3)
(Mines & Lackey, miscellaneous places)

Types of transport of a substance in a fluid (air or water)

NATURAL PROCESSES:

1. *Advection*: Passive entrainment of substance by the carrying fluid



Aerial view of the Rhine-Aare confluence during the tracer experiment July 1989 (photo H. Aschwanden, Landeshydrologie und -geologie, Bern, Switzerland)
(from A. van Mazijk, *One-dimensional approach of transport phenomena of dissolved matter in rivers*, Delft University of technology, Sept. 1996)

Examples

Smokestack fume blown by the wind

Sediments flowing down a river

2. *Diffusion*: Motion with respect to the carrying fluid by random molecular collision

Examples:
spreading of a chemical in still water
spreading of a toxic gas in still air

3. *Turbulent dispersion*: Motion with respect to the carrying fluid by chaotic, turbulent swirls of the fluid motion

Ex*ample*: stirring cream in coffee

4. *Gravitational settling*: Vertical motion with respect to the fluid because of a density difference; particles heavier than fluid sink to the bottom; those lighter than the fluid rise to the top.

Examples:
soil particles settling at the bottom of a lake
oil drops floating on the sea

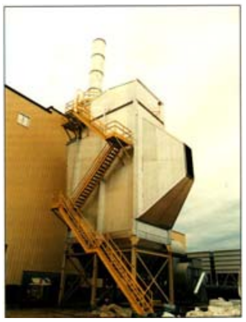
http://www.bubbleology.com/seeps/SeepTarStudy.htm

UNNATURAL PROCESSES

designed to enhance transport when natural processes are too weak

5. *Centrifugal settling* – also called inertial drift: Spinning of the fluid to exert a centrifugal force that acts sideways and can be much stronger than gravity

Example: catching flies on the windshield of a moving car, cyclone dust remover



http://www.ppcesp.com/ppcart.html

6. *Electrostatic settling* – also called *electrostatic drift*: Charging particles and passage through an electric field to force them to migrate out of the fluid toward an electrode where they are collected

Example: electrostatic ash precipitator at a power plant

Notion of mass flux

(Mihelcic & Zimmerman, page 140)

Definition: $J = \frac{\dot{m}}{A} \quad \Leftrightarrow \quad \dot{m} = J\ A$

The flux J of a substance is defined as the amount $\dot{m}$ that is transported per unit area A and per unit time.

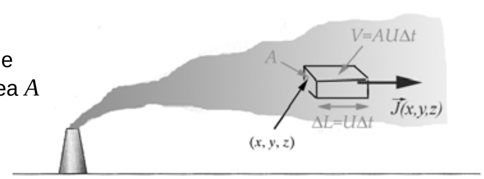


Figure 4.A.1 Flux, $\vec{J}$, is a vector quantity whose value varies with position (x, y, z). The contaminant flux vector points in the direction of transport, and its magnitude is the quantity transported (usually mass or moles) per area per time.

Case of advection:

Over a time interval Δt, fluid moving at velocity U travels a distance $\Delta L = U\Delta t$. For a cross-sectional area A, this defines a volume of size $V = AU\Delta t$.

The amount of substance in that parcel of fluid is

$$V\,C = AU\Delta t\ C$$

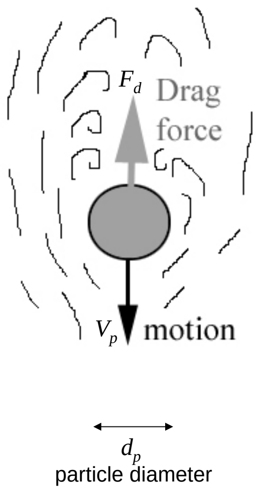
By definition, the flux J of the substance is the amount that passes through per area and per time:

$$J = \frac{\text{amount of substance}}{\text{area} \times \text{time}} = \frac{VC}{A\,\Delta t} = \frac{AU\Delta t\ C}{A\,\Delta t} = \frac{Q}{A}C = U\ C$$

Thus, in the case of **advection**, the flux is the product of the velocity of the entraining fluid and the concentration of the substance.

Drag on falling (rising) particles

(Mihelcic & Zimmerman, page 148)
(Mines & Lackey, page 220 – with added generality)



Most often, particles in air and water are either heavier or lighter. Thus, they fall (settle) or rise with respect to the air or water.

Particles in relative motion with respect to a fluid are subject to a frictional drag force, F_d.

In fluid mechanics, we learn that:

$$F_d = (\text{drag coefficient})(\text{frontal area})\left(\frac{1}{2}\rho_f V_p^2\right)$$

$$= C_d\left(\frac{\pi}{4}d_p^2\right)\left(\frac{1}{2}\rho_f V_p^2\right)$$

$$= \frac{\pi}{8} C_d\ \rho_f\ d_p^2\ V_p^2$$

where
ρ_f = density of fluid
V_p = particle velocity

The drag coefficient is not constant, except for the very large particles that move fast with respect to the fluid.

In general, it is a function of the speed of the particle, measured in terms of the Reynolds number:

$$\mathrm{Re}_p = \frac{\rho_f d_p V_p}{\mu_f}$$

where ρ_f = density of fluid
d_p = particle diameter
V_p = particle velocity
μ_f = fluid viscosity

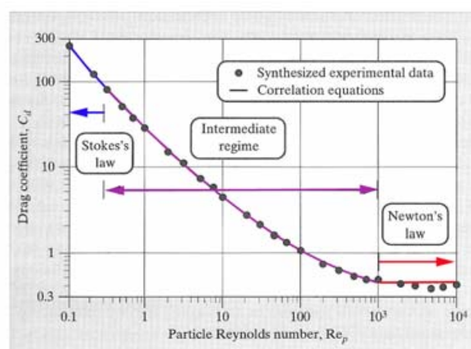


Figure 4.B.1 Drag coefficient as a function of particle Reynolds number for smooth, spherical, nonaccelerating particles in a uniform fluid flow. The experimental data are from Lapple and Shepherd (1940). The correlation equations are described in the text.

Three regimes:

Stokes (slow): $C_d = \frac{24}{\mathrm{Re}_p}$ $\quad \mathrm{Re}_p < 0.3 \longrightarrow F_d = 3\pi\mu_f d_p V_p$

Intermediate: $C_d = \frac{24}{\mathrm{Re}_p}(1 + 0.14\,\mathrm{Re}_p^{0.7})$ $\quad 0.3 < \mathrm{Re}_p < 1000$

Newton (fast): $C_d = 0.445$ $\quad 1000 < \mathrm{Re}_p \longrightarrow F_d = 0.173\rho_f d_p^2 V_p^2$

Correction for the very small particles (diameter < 1 μm = 10^{-6} m):
(Nazaroff & Alvarez-Cohen, page 174)

For very small particles, the fluid molecules may not be that much smaller than the particles, and the fluid flow around the particles begins to appear as if it had a lot of holes through which the particle may more easily pass. This leads to a reduced drag.

The drag force is then reduced (divided) by a factor, called the *Cunningham slip factor*, and denoted C_c:

$$F_d = \frac{\text{uncorrected drag force}}{C_c} = \frac{3\pi\mu_f d_p V_p}{C_c}$$

To get C_c, either use formula or graph:

$$C_c = 1 + \frac{\lambda_g}{d_p}\left[2.51 + 0.80\exp\left(-\frac{0.55 d_p}{\lambda_g}\right)\right]$$

where $\lambda_g = 0.066\ \mu\text{m}$

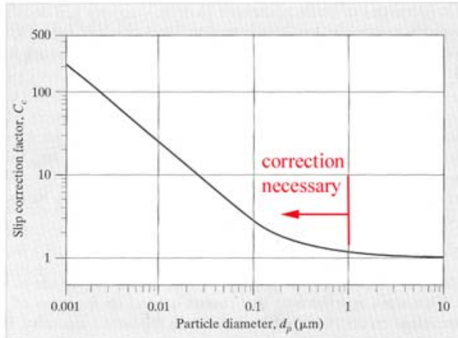


Figure 4.B.2 Slip correction factor for particles in air, assuming that the mean free path is 0.066 μm.

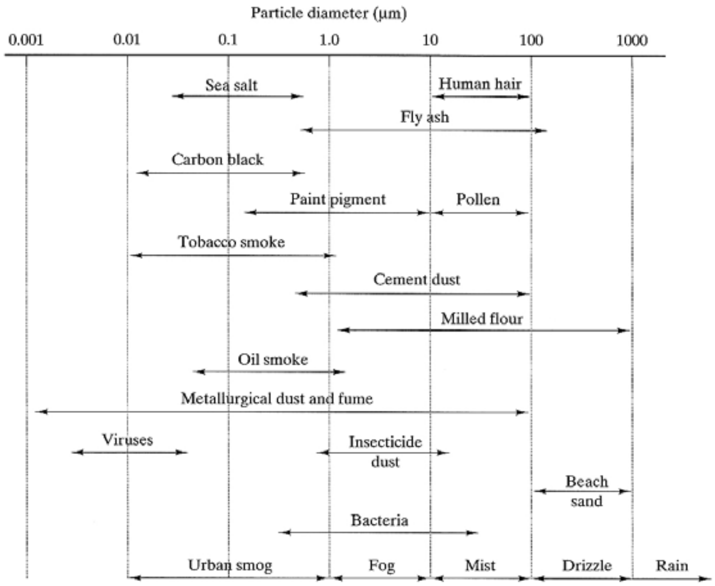
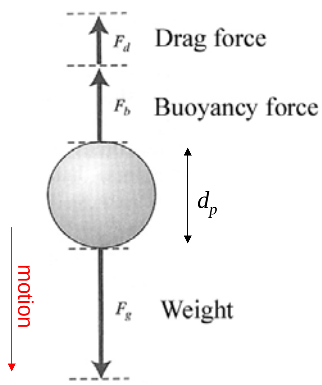


Figure 6-9 Size of common particles
(Source: R. J. Heinsohn & R. L. Kabel, 1999, page 239)

Gravitational settling

(Mihelcic & Zimmerman, page 149)



When moving relatively to a fluid,
a particle is subject to 3 forces:
- its own weight
- a buoyancy force
- a drag force.

After a brief period of acceleration,
a balance is achieved between these 3 forces:

$$F_g = F_b + F_d$$

$$\text{weight of particle: } F_g = m_{\text{particle}}\, g = \rho_p \frac{\pi d_p^3}{6} g$$

$$\text{buoyancy force: } F_b = m_{\text{displaced fluid}}\, g = \rho_f \frac{\pi d_p^3}{6} g$$

$$\text{drag force: } F_d = C_d \frac{\pi}{4} d_p^2 \frac{1}{2} \rho_f V_p^2$$

Note difference: ρ_p = density of material making up the particle, ρ_f = fluid density.

<u>Two extreme (and most common) situations:</u>

Small particles (Stokes' regime $Re_p < 0.3$):

$$V_p = \frac{C_c g d_p^2}{18}\left(\frac{\rho_p - \rho_f}{\mu_f}\right)$$

Large particles (Newton's regime $1000 < Re_p$):

$$V_p = \sqrt{3.0\, g\, d_p \left(\frac{\rho_p - \rho_f}{\rho_f}\right)}$$

Some useful numbers in this context:

$$\text{Air: } \rho_f = 1.20 \text{ kg/m}^3 \qquad \mu_f = 1.8 \times 10^{-5} \text{ kg/(m}\cdot\text{s)}$$

$$\text{Water: } \rho_f = 997 \text{ kg/m}^3 \qquad \mu_f = 1.0 \times 10^{-3} \text{ kg/(m}\cdot\text{s)}$$

$$\text{Sand, grit: } \rho_p = 2650 \text{ kg/m}^3$$

Particle settling in air

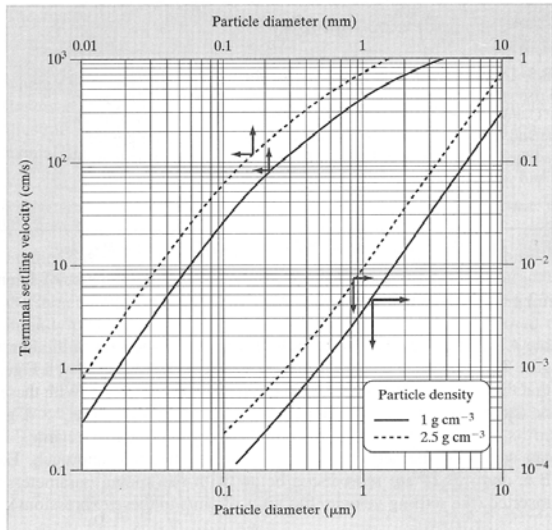


Figure 4.B.4 Terminal settling velocity for rigid spherical particles in air (P = 1 atm, T = 298 K). For the upper pair of curves, read the particle diameter from the upper scale and the settling velocity from the left-hand scale. For the lower pair of curves, read the particle diameter from the lower scale and the settling velocity from the right-hand scale.

Source:
Nazaroff & Alvarez-Cohen, 2001, page 177

Particle settling in water

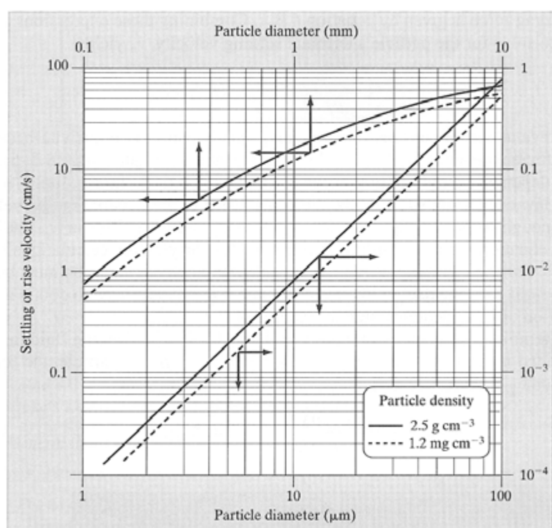


Figure 4.B.5 Terminal settling velocity of rigid spherical particles (ρ = 2.5 g cm^{-3}) and rise velocity of rigid spherical bubbles (ρ = 1.2 mg cm^{-3}) in water at T = 20 °C. For the upper pair of curves, read the particle diameter from the upper scale and the velocity from the left-hand scale. For the lower pair of curves, read the particle diameter from the lower scale and the velocity from the right-hand scale.

Source:
Nazaroff & Alvarez-Cohen, 2001, page 178

EXAMPLE

(Taken from Nazaroff & Alvarez-Cohen, 2001, page 179)

Studies have shown that a white surface becomes noticeably soiled when 0.2% of its area is covered by black particles, such as soot.

Estimate the time required for an initially clean, horizontal surface to appear soiled if it is exposed to an atmosphere containing 10 μg/m^3 of soot particles of diameter 5 μm. Assume that the particles are spherical and have a density of 2.5 g/cm^3.

SOLUTION

Soot particles are small particles. So, assume Stokes drift, but there is no need to apply the Cunningham slip correction factor (particle diameter > 1 μm). The settling speed is:

$$V_p = \frac{C_c g d_p^2}{18}\left(\frac{\rho_p - \rho_f}{\mu_f}\right)$$

$$= \frac{(1)(9.81\ \text{m/s}^2)(5\times10^{-6}\ \text{m})^2}{18}\left(\frac{2.5 - 1.2\times10^{-3}\ \text{g/cm}^3}{1.8\times10^{-4}\ \text{g/cm.s}}\right)$$

$$= 0.189\ \text{cm/s}$$

Determine the mass of each particle

$$m_p = \rho_p \frac{\pi}{6} d_p^3 = (2.5\ \text{g/cm}^3)\frac{\pi}{6}(5\times10^{-4}\ \text{cm})^3 = 1.636\times10^{-10}\ \text{g}$$

Next, determine the number of particles per unit volume of air:

$$C = \frac{\text{mass concentration in the air}}{\text{mass of each particle}} = \frac{10\times10^{-6}\ \text{g/m}^3}{1.636\times10^{-10}\ \text{g/particle}}$$

$$= 61{,}116\ \frac{\text{particles}}{\text{m}^3} = 0.0611\ \frac{\text{particles}}{\text{cm}^3}$$

The flux of falling particles is

$$J = C\,V_p = (0.0611\ \text{particles/cm}^3)(0.189\ \text{cm/s}) = 0.0116\ \frac{\text{particles}}{\text{cm}^2\cdot\text{s}}$$

Each particle has a footprint equal to its cross-sectional area

$$A_p = \frac{\pi}{4} d_p^2 = \frac{\pi}{4}(5\times10^{-4}\ \text{cm})^2 = 1.963\times10^{-7}\ \text{cm}^2$$

Take 1 cm^2 of the surface. It has become soiled when 0.2% of its surface is covered by particles, which is 0.002 cm^2.

This surface coverage necessitates the deposition of a certain number of particles:

$$N = \frac{\text{surface}}{\text{surface per particle}} = \frac{0.002\ \text{cm}^2}{1.963\times10^{-7}\ \text{cm}^2\ /\ \text{particle}} = 10{,}186\ \text{particles}$$

At the rate the particles are falling down, this will take a time equal to

$$\begin{aligned}\frac{10{,}186\ \text{particles per cm}^2\ \text{of surface}}{0.0116\ \text{particles falling per cm}^2\ \text{per second}} &= 881{,}157\ s \\ &= 244.8\ \text{hours} \\ &= 10.2\ \text{days}\end{aligned}$$