

INTRODUCTION TO ENVIRONMENTAL ENGINEERING

HOMEWORK #1 – SOLUTIONS

1. (5 points) (Mihelcic & Zimmerman, page 49, Problem 2.2, 1st half) A water sample contains 10 mg NO₃⁻/L. What is the concentration in (a) ppm on a mass basis, and (b) moles/L?

a)

1 L water = 1 kg water (in water, mg/L = ppm)

$$\frac{10 \text{ mg} / \text{L}}{1 \text{ kg} / \text{L}} = \boxed{10 \text{ ppm as } \text{NO}_3^-}$$

b)

Molecular weight of NO₃⁻ = 14x1 + 16x3 = 62 grams per mole

Thus,

$$\frac{10 \text{ mg}}{\text{L}} \times \frac{1 \text{ g}}{1,000 \text{ mg}} \times \frac{1 \text{ mole}}{62 \text{ g}} = \boxed{1.6 \times 10^{-4} \text{ moles } \text{NO}_3^- / \text{L}}$$

2. (10 points) (Mihelcic & Zimmerman, page 154, Problem 4.1) A pond is used to treat a dilute municipal wastewater before the liquid is discharged into a river. The inflow to the pond has a flow rate of $Q = 4,000 \text{ m}^3/\text{day}$ and a biochemical oxygen demand (BOD) concentration of $C_{in} = 25 \text{ mg/L}$. The volume of the pond is $20,000 \text{ m}^3$. The purpose of the pond is to allow time for the decay of BOD to occur before discharge into the environment. BOD decays in the pond with a first-order rate constant equal to $0.25/\text{day}$. What is the BOD concentration at the outflow of the pond, in units of mg/L ?

$$\frac{dm}{dt} = m_{in} - m_{out} \pm m_{rxn}$$

$$0 = QC_{in} - QC_{out} - VkC_{out}$$

$$0 = 4000 \frac{\text{m}^3}{\text{day}} \times 25 \frac{\text{mg}}{\text{L}} - 4000 \frac{\text{m}^3}{\text{day}} \times (C_{out}) - (20,000 \text{ m}^3) \times (0.25 \frac{1}{\text{day}}) \times (C_{out})$$

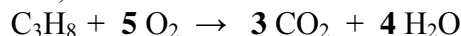
$$\boxed{C_{out} = 11 \frac{\text{mg}}{\text{L}}}$$

3. (10 points) Consider the following reaction representing the combustion of propane:



- (2 points) Balance the equation.
- (2 points) How many moles of oxygen are required to burn 1 mole of propane?
- (3 points) How many grams of oxygen are required to burn 100 g of propane?
- (3 points) Under standard conditions (temperature of 25°C, pressure of 1 atmosphere, and oxygen concentration equal to 20% on a volume basis), what volume of air is required to burn 100 g of propane?

a. (2 points)



b. (2 points)

From the equation above, we immediately note that it takes 5 moles of oxygen (O_2) to burn 1 mole of propane (C_3H_8).

c. (3 points)

To answer this, we first need to determine the molecular weights of both propane and oxygen:

Propane: $\text{C}_3\text{H}_8 \rightarrow 3 \times 12 + 8 \times 1 = 44$ grams/mole

Oxygen: $\text{O}_2 \rightarrow 2 \times 16 = 32$ grams/mole

In 100 grams of propane, there are therefore

$$\frac{100 \text{ grams}}{44 \text{ grams/mole}} = 2.273 \text{ moles of } \text{C}_3\text{H}_8$$

Each mole of propane demands 5 moles of oxygen. The necessary number of moles of oxygen is then:

$$5 \times 2.273 \text{ moles} = 11.364 \text{ moles of } \text{O}_2$$

The corresponding mass of oxygen is:

$$11.364 \text{ moles} \times \frac{32 \text{ grams}}{\text{mole}} = 363.6 \text{ grams of } \text{O}_2$$

d. (3 points)

Oxygen behaves as an ideal gas: $PV = nRT$. This allows us to determine the volume V of oxygen occupied by $n = 11.364$ moles. With:

$T = 25^\circ\text{C} = 298 \text{ K}$ standard (absolute) temperature

$P = 1 \text{ atm}$ standard pressure

we have:

$$V = \frac{nRT}{P} = \frac{(11.364 \text{ moles})(0.08205 \text{ atm} \cdot \text{L/mole} \cdot \text{K})(298 \text{ K})}{(1 \text{ atm})} = 277.9 \text{ L of } \text{O}_2$$

Since oxygen is only 20% of air on a volume basis, the amount of air needed is

$$\frac{277.9 \text{ L of } O_2}{0.20} = 1,389 \text{ L of air} = 1.39 \text{ m}^3 \text{ of air}$$

4. (10 points) (Mihelcic & Zimmerman, page 155, Problem 4.13) A 1.0×10^6 gallon reactor is used in a sewage-treatment plant. The influent concentration is 100 mg/L, the effluent concentration is 25 mg/L, and the flow rate through the reactor is 500 gal/min.

(a) What is the first-order rate constant for decay of BOD in the reactor? Assume that the reactor can be modeled as a CMFR. Report your answer in units of per hour.

(b) Assume that the reactor should be modeled as a PFR with first-order decay, *not* as a CMFR. In that case, what must the first-order decay rate constant be within the PFR reactor?

(c) It has been determined that the outlet concentration is too high, so the residence time in the reactor must be doubled. Assuming all other variables remain constant, what must be the volume of the new CMFR?

a)

We are given:

$V = 1.0 \times 10^6$ gal, $C_{in} = 100$ mg/L, $C_{out} = 25$ mg/L, $Q = 500$ gal/min.

Assuming steady state conditions, the mass balance on BOD in the CMFR is given as follows:

$$\frac{dm}{dt} = m_{in} - m_{out} - m_{rxn}$$

$$0 = QC_{in} - QC_{out} - Vkc$$

$$C = C_{out}$$

$$k = \frac{QC_{in} - QC_{out}}{VC_{out}} = \frac{Q(C_{in} - C_{out})}{VC_{out}} = \frac{(500 \text{ gal / min})(100 \text{ mg / L} - 25 \text{ mg / L})}{(1.0 \times 10^6 \text{ gal})(25 \text{ mg / L})} \times 60 \frac{\text{min}}{\text{h}}$$

$$\boxed{k = 0.090 / \text{h}}$$

b)

$$C_t = C_{in} \times e^{-kt}$$

$$\frac{dC}{dt} = -k C$$

$$C_{out} = C_{in} \times e^{-k\theta}$$

$$\theta = \frac{V}{Q} = \frac{1.0 \times 10^6 \text{ gal}}{500 \text{ gal / min}} = 2 \times 10^3 \text{ min}$$

$$25 \text{ mg / L} = 100 \text{ mg / L} \times e^{-k(2 \times 10^3 \text{ min})}$$

$$k = 6.93 \times 10^{-4} / \text{min} \times 60 \text{ min / h} = \boxed{0.042 / \text{h}}$$

c)

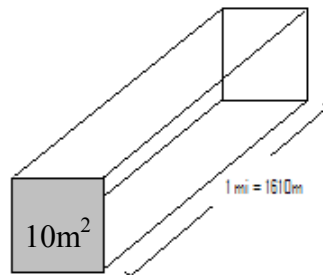
$\theta = V/Q \rightarrow$ if θ doubles and Q remains constant, V must double.

$$2 \times (1.0 \times 10^6 \text{ gal}) = \boxed{2 \times 10^6 \text{ gal}}$$



5. (10 points) (From another textbook) A typical motorcycle emits 20 g of carbon monoxide (CO) per mile, and the tailpipe emission diffuses in a 10-m² cross-section behind the cyclist (perpendicular to the direction of travel). What is the maximum number of motorcycles that can be in a group going one behind another along a street before the CO concentration exceeds to air quality standard of 9.0 ppm? (Assume complete mixing in the 10-m² cross-section and that ambient air is at 20°C and 1 atmosphere.)

Take 1 mile of travel
and do budget.



$$16100 \text{ m}^3 \text{ air} \times \frac{1000 \text{ L}}{\text{m}^3} \times \frac{1.203 \text{ g}}{\text{L}} \times \frac{1 \text{ mol air}}{28.9 \text{ g air}} = 6.70183 \times 10^5 \text{ moles of air}$$

$$6.70183 \times 10^5 \text{ moles air} \times \frac{9 \text{ mol CO}}{1 \times 10^6 \text{ mol air}} = 6.03165 \text{ moles of CO}$$

$$6.03165 \text{ moles CO} \times \frac{28 \text{ g CO}}{1 \text{ mol CO}} \times \frac{1 \text{ motorcycle}}{20 \text{ g CO}} = 8.44 \text{ motorcycles}$$

The maximum number of motorcycles permitted on the road at one time is thus **8**.
(9 motorcycles exceed the acceptable limit.)