

Dimensions and Units of Measure

(Mines & Lackey, Section 2.2)

Every physical, chemical or biological quantity possesses a dimension.

Examples:

a distance is a length, a speed a length per time, a volume a length cubed, a density is a mass per volume, a force a mass times length per time squared.

The dimension of a quantity exists irrespective of the system of units used to ascribe a numerical value to the quantity.

In the United States, two systems of units are commonly used:

Dimension	Length	Mass	Time	Temperature	Force	Energy
Metric system	meter (m)	kilogram (kg)	second (s)	deg Celsius (°C)	newton (N)	joule (J)
US Customary System	foot (ft)	pound (lb)	second (s)	deg Fahrenheit (°F)	pound-force (lb _f)	Br. thermal unit (BTU)

In many situations, ratios may be constructed to compare two quantities of the same dimension. The result is a dimensionless number, the value of which is independent of the unit system being used.

Density

(Mines & Lackey, Section 2.3.1)

The concept of density exists to answer the question:

How heavy or light is a substance?

Definition: The density of a substance is its mass per volume.

Notation: ρ , "rho", the Greek letter for "r".

The dimension of density is thus mass per length cubed (M/L³).

Depending on the system of units used, density is expressed as follows:

Metric System: in kg/m³.

US Customary System: in lb/ft³.

Example: Water at 10°C (= 50°F), $\rho = 999.7 \text{ kg/m}^3 = 62.42 \text{ lb/ft}^3$.

Concentration of substance inside of another

(Mines & Lackey, Section 2.3.2 + added elements)

The concept of concentration exists to answer the question:

How much of the “stuff” is there?

Definition: The concentration of a substance is the “amount” of it per “amount” of containing material (air, water, soil).

It can be expressed in various units.

If the containing medium is air for example:

C_A = mass of A / volume of air	used with mass balances
$[A]$ = moles of A / volume of air	used with chemical reactions
m_A = mass of A / mass of A + air	used when air pressure varies
x_A = moles of A / moles of air	idem
P_A = partial pressure of A (< atmospheric pressure)	used for air-water exchange

Unit conversion

It is often necessary to switch units, for example, to pass from a chemical reaction (in which amounts are most naturally expressed in moles) to a mass budget (in which amounts are most naturally expressed in grams).

Rule 1:

$$\text{Mass in grams} = \text{Molecular weight} \times \text{Number of moles}$$

where

$$\text{Molecular weight} = \sum \text{Atomic weights}$$

Examples:

H₂O: MW = 2x1 + 1x16 = 2 + 16 = 18 grams per mole

CO₂: MW = 1x12 + 2x16 = 12 + 32 = 44 grams per mole

NaNO₃: MW = 1x23 + 1x14 + 3x16 = 23 + 14 + 48 = 85 grams per mole

H₂SO₄: MW = 2x1 + 1x32 + 4x16 = 2 + 32 + 64 = 98 grams per mole.

**Atomic weights
most commonly used in environmental engineering**

Hydrogen	H	1	
Carbon	C	12	(14 is for radioactive form of C !)
Nitrogen	N	14	
Oxygen	O	16	
Sodium	Na	23	
Phosphorus	P	31	
Sulfur	S	32	
Chlorine	Cl	35.45	
Calcium	Ca	40	

It is helpful to memorize the preceding numbers.

For other elements, see the Periodic Table of Elements, on the next slide

Periodic Table of Elements

(see also Mines & Lackey, page 33)

Note: Round numbers when they are close to being integers.

Examples: Hydrogen = H = 1; Carbon = C = 12; Oxygen = O = 16
but Chlorine = Cl = 35.45.

hydrogen 1 H 1.00794(7)																		helium 2 He 4.002602(2)																																													
lithium 3 Li 6.941(2)		beryllium 4 Be 9.012183(2)																		boron 5 B 10.811(7)		carbon 6 C 12.0107(8)		nitrogen 7 N 14.0064(2)		oxygen 8 O 15.9994(3)		fluorine 9 F 18.9984032(3)		neon 10 Ne 20.1797(6)																																	
sodium 11 Na 22.98976928(2)		magnesium 12 Mg 24.30409(4)		aluminum 13 Al 26.9815386(8)		silicon 14 Si 28.0855(8)		phosphorus 15 P 30.97376199(8)		sulfur 16 S 32.06(5)		chlorine 17 Cl 35.45(2)		argon 18 Ar 39.948(1)		potassium 19 K 39.0983(1)		calcium 20 Ca 40.078(4)		scandium 21 Sc 44.955910(9)		titanium 22 Ti 47.867(1)		vanadium 23 V 50.9415(1)		chromium 24 Cr 51.9961(6)		manganese 25 Mn 54.938044(1)		iron 26 Fe 55.845(2)		cobalt 27 Co 58.933195(5)		nickel 28 Ni 58.6934(4)		copper 29 Cu 63.546(3)		zinc 30 Zn 65.38(2)		gallium 31 Ga 69.723(1)		germanium 32 Ge 72.64(1)		arsenic 33 As 74.92160(3)		selenium 34 Se 78.96(2)		bromine 35 Br 79.904(1)		krypton 36 Kr 83.796(2)													
rubidium 37 Rb 85.4678(3)		strontium 38 Sr 87.62(1)		yttrium 39 Y 88.90585(2)		zirconium 40 Zr 91.224(2)		niobium 41 Nb 92.90638(2)		molybdenum 42 Mo 95.96(2)		technetium 43 Tc [98]		ruthenium 44 Ru 101.07(2)		rhodium 45 Rh 102.90550(2)		palladium 46 Pd 106.42(1)		silver 47 Ag 107.8682(2)		cadmium 48 Cd 112.411(8)		indium 49 In 114.818(3)		tin 50 Sn 118.710(2)		antimony 51 Sb 121.760(1)		tellurium 52 Te 127.60(3)		iodine 53 I 126.90447(3)		xenon 54 Xe 131.29(3)																													
cesium 55 Cs 132.9054519(2)		barium 56 Ba 137.327(7)		lanthanum 57 La 138.90471(3)		cerium 58 Ce 140.12(1)		praseodymium 59 Pr 140.90766(2)		neodymium 60 Nd 144.24(1)		promethium 61 Pm [145]		samarium 62 Sm 150.36(2)		europium 63 Eu 151.964(1)		gadolinium 64 Gd 157.25(3)		terbium 65 Tb 158.92535(2)		dysprosium 66 Dy 162.50(3)		holmium 67 Ho 164.93033(2)		erbium 68 Er 167.259(3)		thulium 69 Tm 168.93047(3)		ytterbium 70 Yb 173.054(8)		lutetium 71 Lu 174.967(1)		hafnium 72 Hf 178.49(2)		tantalum 73 Ta 180.94788(1)		tungsten 74 W 183.84(1)		rhenium 75 Re 186.207(1)		osmium 76 Os 190.23(3)		iridium 77 Ir 192.222(1)		platinum 78 Pt 195.078(2)		gold 79 Au 196.966569(2)		mercury 80 Hg 200.59(2)		thallium 81 Tl 204.3833(2)		lead 82 Pb 207.2(1)		bismuth 83 Bi 208.98039(2)		polonium 84 Po [209]		astatine 85 At [210]		radon 86 Rn [222]	
francium 87 Fr [223]		radium 88 Ra [226]		actinium 89 Ac [227]		thorium 90 Th 232.0377(2)		protactinium 91 Pa 231.03688(2)		uranium 92 U 238.02891(3)		neptunium 93 Np [237]		plutonium 94 Pu [244]		americium 95 Am [243]		curium 96 Cm [247]		berkelium 97 Bk [247]		californium 98 Cf [251]		einsteinium 99 Es [252]		fermium 100 Fm [257]		mendelevium 101 Md [258]		nobelium 102 No [259]		lawrencium 103 Lr [260]		rutherfordium 104 Rf [261]		dubnium 105 Db [262]		seaborgium 106 Sg [263]		bohrium 107 Bh [264]		hassium 108 Hs [265]		meitnerium 109 Mt [266]		darmstadtium 110 Ds [267]		roentgenium 111 Rg [268]		unbinilium 112 Uub [269]		ununilium 113 Uut [270]		unununium 114 Uuq [271]		ununbium 115 Uup [272]		ununtrium 116 Uuh [273]		ununquadium 117 Uus [274]		ununpentium 118 Uuo [276]	

http://www.webelements.com/nexus/files/WEB_ELEM.pdf

Rule 2:

Pressure of a gas is determined from the ideal-gas law
(Mines & Lackey, Section 2.3.2, pages 23 to 26)

$$P_A V = n_A RT$$

where P_A = (partial) pressure of A, in atm (atmosphere)
 V = volume occupied, in m^3 (entire volume, even if shared with other gases)
 n_A = number of moles of A in that volume
 R = universal constant = $8.206 \times 10^{-5} \text{ atm} \cdot m^3 / (\text{mol} \cdot K)$
 T = absolute temperature, in degrees Kelvin (K)

Recall:

Absolute temperature (K) = temperature in degrees Celsius ($^{\circ}C$) + 273.15

When several gases occupy a common volume (mixture),
their partial pressures simply add up to the total pressure:

"Dalton's Law"
(Miles & Lackey, p. 53)

$$P_{\text{total}} = P_A + P_B + P_C + \dots = (n_A + n_B + n_C + \dots) \frac{RT}{V}$$

← shared volume

Properties of air

Apply ideal-gas law to air. Thus,

At standard pressure ($P = 1 \text{ atm}$) and temperature ($T = 15^{\circ}C = 288.15 \text{ K}$),
One mole ($n = 1 \text{ mol}$) of air occupies a volume V equal to

$$V = \frac{nRT}{P} = \frac{(1 \text{ mol})(8.205 \times 10^{-5} \text{ atm} \cdot m^3/\text{mol K})(288.15 \text{ K})}{(1 \text{ atm})}$$
$$= 0.02364 m^3 = 23.64 \text{ L (liters)} = 6.25 \text{ gallons}$$

Also,

Air = mixture of 79% nitrogen + 21% oxygen

$$\begin{aligned} MW_{\text{air}} &= (0.79) MW_{\text{nitrogen}} + (0.21) MW_{\text{oxygen}} \\ &= (0.79)(2 \times 14) + (0.21)(2 \times 16) \\ &= 22.12 + 6.72 = 28.84 \text{ grams per mole} \end{aligned}$$

Actually, the value is **28.95 g/mol** because of rare gases (heavier).

This leads to:

$$\begin{aligned} 1 / (23.64 \text{ L/mol}) &= 0.0423 \text{ mol/L} = \mathbf{42.3 \text{ mol/m}^3} \\ (28.95 \text{ g/mol}) / (23.64 \text{ L/mol}) &= \mathbf{1.22 \text{ g/L}} \quad \text{at ambient pressure and temperature} \end{aligned}$$

Properties of water

Similar numbers for water (an incompressible liquid) are:

$$\text{H}_2\text{O} \rightarrow \text{MW} = 2 \times 1 + 1 \times 16 = 18 \rightarrow \mathbf{18.0 \text{ g/mol}}$$

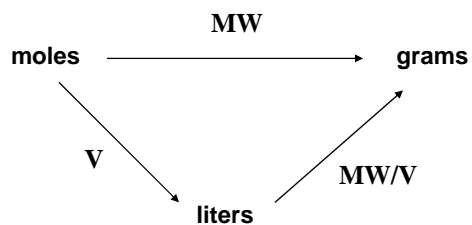
$$\text{Density} = \mathbf{999 \text{ g/L}} \quad (\text{Think: 1 kg per liter})$$

Combine the above:

$$(999 \text{ g/L}) / (18.0 \text{ g/mol}) = \mathbf{55.5 \text{ mol/L}}$$

Summary of unit conversion

1 mole weighs **MW** grams and occupies **V** liters



moles to grams: multiply by **MW**

moles to liters: multiply by **V**

liters to grams: multiply by **MW/V**

Common abbreviations

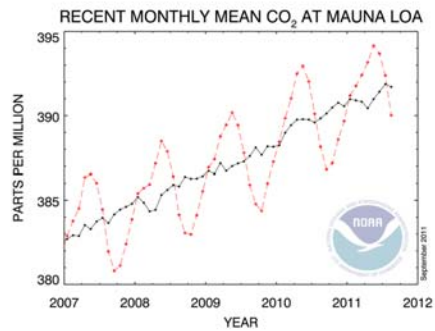
%	percent	1 part in 100
‰	per mil	1 part in 1000
ppm	part per million	1 part in 10^6
ppb	part per billion	1 part in 10^9
ppt	part per trillion	1 part in 10^{12}

where the “part” usually stands for “mole”.

Example:

Today's carbon dioxide concentration in the atmosphere is reported to be about 390 ppm.

This means that there are 390 moles of CO_2 per million moles of air.



In-class exercise

Knowing that the total mass of the atmosphere is
 4.99×10^{15} metric tons

And that the current CO_2 concentration is
390 ppm,

How many tons of CO_2 are there in the atmosphere?

Then, what is the partial pressure of CO_2 in the atmosphere?

How much weight has been added on your shoulders since the CO_2 concentration rose from its pre-industrial level of 270 ppm?

Flow rate

(Mines & Lackey, Section 2.3.3, page 26)

The concept of flow rate (or “flowrate” in one word) exists to answer the question:

How much is flowing by? As in a discharge pipe, river, wind.

Depending on the choice, there are two ways of quantifying this: mass per time and volume per time. Of the two, volume per time is the most common.

Definition: Flow rate is volume (or mass) per time.

Notation: Q (alternatively Q_m if one deals with mass per time instead)

Often used relation: $Q = A u$, product of cross-sectional area A and fluid velocity u

The dimension of a flow rate is thus length cubed per time (L^3/T).

Depending on the system of units used, flow rate is expressed as follows:

Metric System: in m^3/s (or kg/s).

US Customary System: in ft^3/s (or lb/s)

Common in US: gallons per minute (gpm), million gallons per day (Mgd).

Residence time

(Mines & Lackey, Section 2.3.4, pages 27-28)

The concept of “residence time” (alternatively called “detention time” or “retention time”) exists to answer the question:

How long does the “stuff” stays “there”?

Definition: Residence time = Containing volume divided by flow rate.

Notation: θ , “theta”, the Greek letter for “th”.

$$\theta = \frac{V}{Q}$$

The dimension is time (T) because V/Q has dimensions of $L^3/(L^3/T) = T$.

While the standard unit is the second, typical residence times in the environment are measured in days, weeks, months or even years, depending on the size of the containing volume (ex. small pond to large lake).

Example of residence time

(Mihelcic & Zimmerman, Section 4.1.6, page 128)

$$\theta = V / Q$$

The Great Lakes

Age of waters at Niagara Falls:

Up to 182 yrs in Lake Erie
+ 161 yrs in Lake Huron
+ 67 yrs in Lake Superior
= up to 410 years.

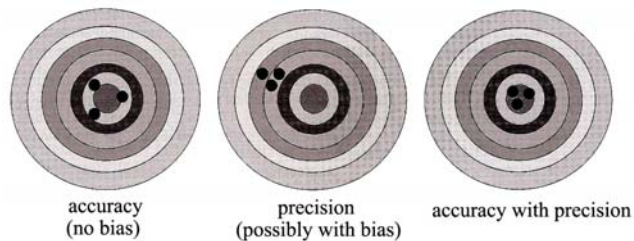


We are now in 2011.
Subtracting 410 years brings us back to 1601 AD.
What was happening in the world then?

Precision, Bias, Accuracy

(Mines & Lackey, section 2.4)

In engineering generally and especially in environmental matters, no quantity can be known perfectly well. There will always be some error attached to the measurement or to the calculated value.



Precision indicates how well the measurement is made. Repeated measurements then yield very similar values, but there may be a **bias** in the measuring technique.

Accuracy describes how close a measurement actually gets to the true value.