Dimensions and Units of Measure

(Mines & Lackey, Section 2.2)

Every physical, chemical or biological quantity possesses a dimension.

Examples:
a distance is a length, a speed a length per time, a volume a length cubed,
a density is a mass per volume, a force a mass times length per time squared.

The dimension of a quantity exists irrespective of the system of units used to ascribe a numerical value to the quantity.

In the United States, two systems of units are commonly used:

Dimension	Length	Mass	Time	Temperature	Force	Energy
Metric system	meter (m)	kilogram (kg)	second (s)	deg Celsius (^{o}C)	newton (N)	joule (J)
US Customary System	foot (ft)	pound (lb)	second (s)	deg Fahrenheit (^{o}F)	pound-force (lb_f)	Br. thermal unit (BTU)

In many situations, ratios may be constructed to compare two quantities of the same dimension. The result is a dimensionless number, the value of which is independent of the unit system being used.

Density

(Mines & Lackey, Section 2.3.1)

The concept of density exists to answer the question:

How heavy or light is a substance?

Definition: The density of a substance is its mass per volume.

Notation: ρ, “rho”, the Greek letter for “r”.

The dimension of density is thus mass per length cubed (M/L^3).

Depending on the system of units used, density is expressed as follows:
Metric System: in kg/m^3.
US Customary System: in lb/ft^3.

Example: Water at 10^{o}C (= 50^{o}F), r = 999.7 kg/m^3 = 62.42 lb/ft^3.

Concentration of substance inside of another

(Mines & Lackey, Section 2.3.2 + added elements)

The concept of concentration exists to answer the question:

How much of the "stuff" is there?

Definition: The concentration of a substance is the "amount" of it per "amount" of containing material (air, water, soil).

It can be expressed in various units.
If the containing medium is air for example:

C_A = mass of A / volume of air	used with mass balances
$[A]$ = moles of A / volume of air	used with chemical reactions
m_A = mass of A / mass of A + air	used when air pressure varies
x_A = moles of A / moles of air	idem
P_A = partial pressure of A (< atmospheric pressure)	used for air-water exchange

Unit conversion

It is often necessary to switch units, for example, to pass from a chemical reaction (in which amounts are most naturally expressed in moles) to a mass budget (in which amounts are most naturally expressed in grams).

Rule 1:

Mass in grams = Molecular weight x Number of moles

where

Molecular weight = Σ Atomic weights

Examples:

H_2O: MW = 2x1 + 1x16 = 2 + 16 = 18 grams per mole
CO_2: MW = 1x12 + 2x16 = 12 + 32 = 44 grams per mole
$NaNO_3$: MW = 1x23 + 1x14 + 3x16 = 23 + 14 + 48 = 85 grams per mole
H_2SO_4: MW = 2x1 + 1x32 + 4x16 = 2 + 32 + 64 = 98 grams per mole.

Atomic weights
most commonly used in environmental engineering

Hydrogen	**H**	1	
Carbon	**C**	12	(14 is for radioactive form of C !)
Nitrogen	**N**	14	
Oxygen	**O**	16	
Sodium	**Na**	23	
Phosphorus	**P**	31	
Sulfur	**S**	32	
Chlorine	**Cl**	35.45	
Calcium	**Ca**	40	

It is helpful to memorize the preceding numbers.

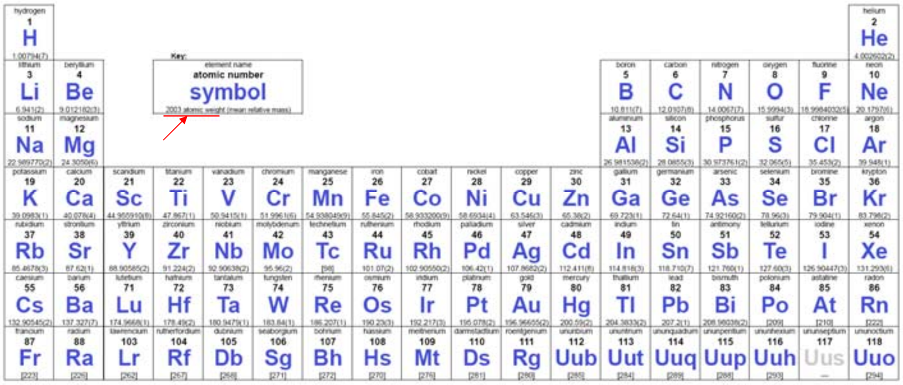
For other elements, see the Periodic Table of Elements, on the next slide

Periodic Table of Elements

(see also Mines & Lackey, page 33)

***Note*: Round numbers when they are close to being integers.**

***Examples*: Hydrogen = H = 1; Carbon = C = 12; Oxygen = O = 16**
but Chlorine = Cl = 35.45.



Key:
element name
atomic number
symbol
2003 atomic weight (mean relative mass)

1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	71 Lu	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	103 Lr	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Uub	113 Uut	114 Uuq	115 Uup	116 Uuh	117 Uus	118 Uuo

http://www.webelements.com/nexus/files/WEB_ELEM.pdf

Rule 2:

Pressure of a gas is determined from the ideal-gas law
(Mines & Lackey, Section 2.3.2, pages 23 to 26)

$$\boxed{P_A V = n_A RT}$$

where P_{A} = (partial) pressure of A, in atm (atmosphere)
V = volume occupied, in m^3 (entire volume, even if shared with other gases)
n_{A} = number of moles of A in that volume
R = universal constant = 8.206×10^{-5} atm • m^3 / (mol • K)
T = absolute temperature, in degrees Kelvin (K)

Recall:
Absolute temperature (K) = temperature in degrees Celsius ($^{\text{o}}$C) + 273.15

When several gases occupy a common volume (mixture), their partial pressures simply add up to the total pressure:

"Dalton's Law"
(Miles & Lackey, p. 53)

$$P_{\text{total}} = P_A + P_B + P_C + \ldots = (n_A + n_B + n_C + \ldots)\frac{RT}{V}$$

shared volume

Properties of air

Apply ideal-gas law to air. Thus,

At standard pressure (P = 1 atm) and temperature (T = 15$^{\text{o}}$C = 288.15 K),
One mole (n = 1 mol) of air occupies a volume V equal to

$$V = \frac{nRT}{P} = \frac{(1\,\text{mol})(8.205 \times 10^{-5}\ \text{atm} \cdot \text{m}^3/\text{mol K})(288.15\,\text{K})}{(1\,\text{atm})}$$

$$= 0.02364\,\text{m}^3 = 23.64\,\text{L (liters)} = 6.25\,\text{gallons}$$

Also,

Air = mixture of 79% nitrogen + 21% oxygen
MW_{air} = (0.79) $\text{MW}_{\text{nitrogen}}$ + (0.21) $\text{MW}_{\text{oxygen}}$
= (0.79)(2 x 14) + (0.21)(2 x 16)
= 22.12 + 6.72 = 28.84 grams per mole
Actually, the value is **28.95 g/mol** because of rare gases (heavier).

This leads to:

1 / (23.64 L/mol) = 0.0423 mol/L = **42.3 mol/m^3**
(28.95 g/mol) / (23.64 L/mol) = **1.22 g/L** at ambient pressure and temperature

Properties of water

Similar numbers for water (an incompressible liquid) are:

H_2O → MW = 2x1 + 1x16 = 18 → **18.0 g/mol**

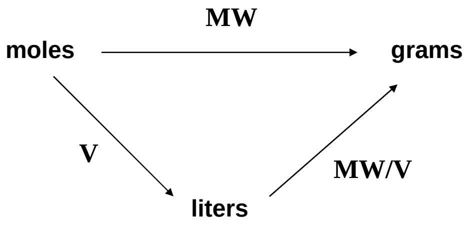
Density = **999 g/L** (Think: 1 kg per liter)

Combine the above:

(999 g/L) / (18.0 g/mol) = **55.5 mol/L**

Summary of unit conversion

1 mole weighs **MW** grams and occupies **V** liters



MW

moles

grams

V

MW/V

liters

moles to grams: multiply by **MW**

moles to liters: multiply by **V**

liters to grams: multiply by **MW/V**

Common abbreviations

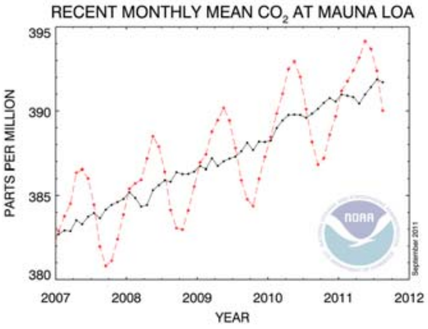
%	percent	1 part in 100
‰	per mil	1 part in 1000
ppm	part per million	1 part in 10^6
ppb	part per billion	1 part in 10^9
ppt	part per trillion	1 part in 10^{12}

where the “part” usually stands for “mole”.

Example:

Today’s carbon dioxide concentration in the atmosphere is reported to be about 390 ppm.

This means that there are 390 moles of CO_2 per million moles of air.



In-class exercise

Knowing that the total mass of the atmosphere is
4.99 x 10^{15} metric tons

And that the current CO_2 concentration is
390 ppm,

How many tons of CO_2 are there in the atmosphere?

Then, what is the partial pressure of CO_2 in the atmosphere?

How much weight has been added on your shoulders since the CO_2 concentration rose from its pre-industrial level of 270 ppm?

Flow rate

(Mines & Lackey, Section 2.3.3, page 26)

The concept of flow rate (or “flowrate” in one word) exists to answer the question:

How much is flowing by? As in a discharge pipe, river, wind.

Depending on the choice, there are two ways of quantifying this: mass per time and volume per time. Of the two, volume per time is the most common.

Definition: Flow rate is volume (or mass) per time.

Notation: Q (alternatively Q_m if one deals with mass per time instead)

Often used relation: $Q = A\ u$, product of cross-sectional are A and fluid velocity u

The dimension of a flow rate is thus length cubed per time (L^3/T).

Depending on the system of units used, flow rate is expressed as follows:
- Metric System: in m^3/s (or kg/s).
- US Customary System: in ft^3/s (or lb/s)
- Common in US: gallons per minute (gpm), million gallons per day (Mgd).

Residence time

(Mines & Lackey, Section 2.3.4, pages 27-28)

The concept of “residence time” (alternatively called “detention time” or “retention time”) exists to answer the question:

How long does the “stuff” stays “there?

Definition: Residence time = Containing volume divided by flow rate.

Notation: θ, “theta”, the Greek letter for “th”.

$$\theta = \frac{V}{Q}$$

The dimension is time (T) because V/Q has dimensions of $L^3/(L^3/T) = T$.

While the standard unit is the second, typical residence times in the environment are measured in days, weeks, months or even years, depending on the size of the containing volume (ex. small pond to large lake).

Example of residence time

(Mihelcic & Zimmerman, Section 4.1.6, page 128)

$$\theta = V / Q$$

The Great Lakes

Age of waters at Niagara Falls:

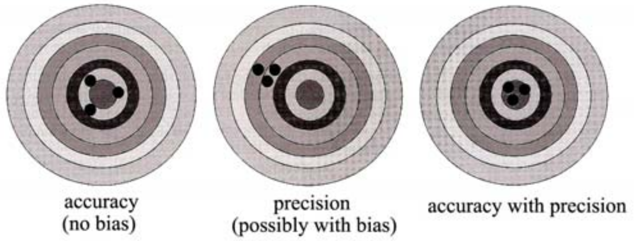
Up to 182 yrs in Lake Erie
+ 161 yrs in Lake Huron
+ 67 yrs in Lake Superior
= up to 410 years.

We are now in 2011.
Subtracting 410 years brings us back to 1601 AD.
What was happening in the world then?

Precision, Bias, Accuracy

(Mines & Lackey, section 2.4)

In engineering generally and especially in environmental matters, no quantity can be known perfectly well. There will always be some error attached to the measurement or to the calculated value.



Precision indicates how well the measurement is made. Repeated measurements then yield very similar values, but there may be a **bias** in the measuring technique.

Accuracy describes how close a measurement actually gets to the true value.