

Chapter 13

Atmospheric Boundary Layer

SUMMARY: This chapter considers the physics of the lowest portion of the atmosphere, in which we live and breathe. The central processes are wind stirring (mechanical turbulence) and diurnal convection (thermal turbulence).

13.1 The Lower Atmosphere

The lowest portion of the atmosphere is vital to us, for it is where we live and breathe. It is also where all our gaseous emissions are discharged. Thus, air quality is most important in the lowest level of the atmosphere, and the physics affecting it form an essential element of Environmental Fluid Mechanics.

Entire books and textbooks have been devoted to the topics of the *Atmospheric Boundary Layer* (for ex. Stull, 1988; Sorbjan, 1989; Garratt, 1992; Kaimal and Finnigan, 1994) and *Air Pollution Meteorology* (for ex. Lyons and Scott, 1990; Boubel et al., 1994; Scorer, 2002). The interested reader will find in these a comprehensive treatment of the topic, well beyond what this single chapter can expose.

The lowest atmosphere is far from being a simple system. Its physics include a diurnal component (typically convection during the day and stratification at night), complications due to complex terrain (surface elements such as buildings, forests, hills and mountains) and larger weather events (replacement of air masses by prevailing winds; clouds and precipitation).

Whereas the atmosphere is more than 100 km thick, weather systems, including cyclones, anticyclones, storms and hurricanes, hardly occupy more than the bottom 10 km, a layer called the *troposphere*. Unlike the quiet *stratosphere*, the layer immediately above (10–50 km altitude), the troposphere is in a permanent state of turmoil. Its ubiquitous instabilities, while challenging the fluid dynamicist, are highly beneficial to us, on two levels. First, the instabilities

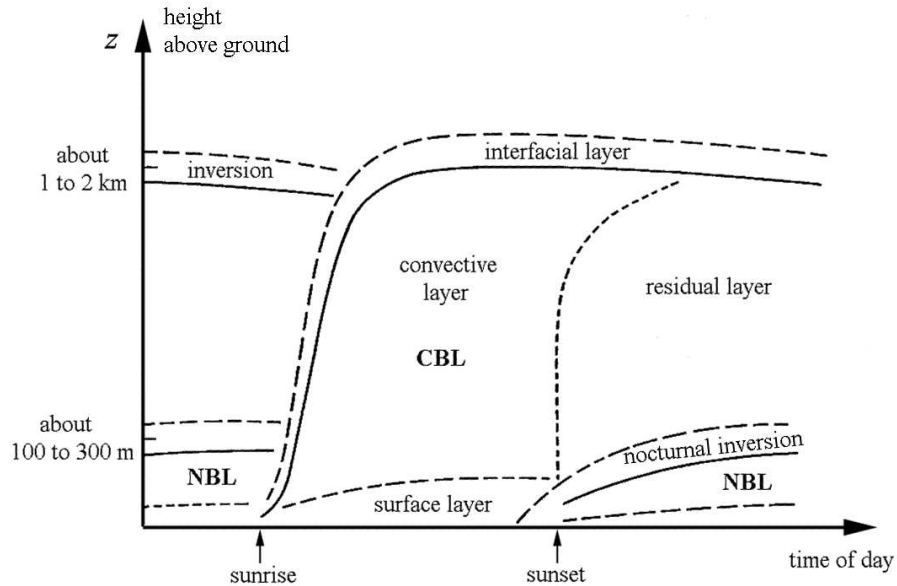


Figure 13.1: Typical evolution of the atmospheric boundary layer (ABL) over the course of a day over land and under clear skies. At sunrise, heating from below sets to a convective boundary layer (CBL), while at sunset heat loss to space terminates convection and creates a thin nocturnal boundary layer (NBL). [Adapted from Garratt, 1992]

create weather patterns, which cause precipitation, water the crops and provide a freshwater supply to us on land. Second, turbulent mixing generates dispersion and dilution of our pollutants.

Within the troposphere, closest to the ground, lies the *Atmospheric Boundary Layer* (ABL). It is about 1 km thick and forms the layer where the atmosphere feels the contact with the ground surface, land or sea. The surface-air interaction occurs in two primary forms: mechanical and thermal. The mechanical contact arises from the friction exerted by the wind against the ground surface; this friction causes the wind to be sheared and creates turbulence. In the absence of thermal processes, i.e. when the ABL is said to be *neutral*, we expect a logarithmic velocity profile $u(z)$ (see Section 8.2) characterized by the friction velocity u_* and the roughness height z_o . The friction velocity is intimately related to the level of turbulence in the lowest atmosphere.

The thermal contact between the lower atmosphere and the ground surface has its origin in the solar radiation. Sun light is electromagnetic radiation in the visible range, to which the atmosphere is largely transparent. (We see through the air over fairly long distances.) Therefore, much of the solar radiation traverses the atmosphere and reaches the surface. Land surface, by contrast, is

opaque, while water in the sea or lakes is somewhat transparent but much less than the air, and consequently most of the solar radiation is absorbed immediately below the earth surface. This surface, in turn, heats up and radiates heat away, otherwise its temperature would rise indefinitely. But, because the earth surface is much cooler than the sun, it emits its heat in much lower frequencies of the electromagnetic wave spectrum, in the infrared range. This radiation, invisible to us but feeling like heat, is emitted upward into the atmosphere and, although some portion escapes to space, much is retained in the atmosphere because of absorption by water vapor, carbon dioxide and other gases. This heat retention is called the *greenhouse effect*. Leaving climatic implications aside, we note that the atmosphere is paradoxically heated from *below*, despite the obvious fact that the original source of heat – the sun – is *above* it. In some sense, the atmosphere is not unlike water in a kettle above the stove. Both are heated from below and in a state of convection. The quantities characterizing this state of convection are the heat flux Q (in W/m^2) and the thickness h of the convection.

A crucial aspect of the thermal contact between air and its underlying surface is its diurnal intermittency: The sun shines during the day and is absent at night. Absence of convection during nighttime creates vertical stratification, which daytime convection proceeds to erode (Figure 13.1). The efficacy of erosion during the day depends in part on the amount of cloud cover. A thick layer of clouds can intercept much of the sunlight and reduce the amount of heat delivered to ground level. When convection occurs, the mixing motions typically engulf the lowest 1 to 2 km of the atmosphere. This height, denoted h , is what is commonly used as the thickness of the ABL.

Figure 13.1 traces the intermittently convective behavior of the ABL under clear skies. The night generally ends with a shallow *nocturnal boundary layer* (NBL), in which mixing is caused by wind friction. The thickness of this layer depends on the prevailing wind velocity as well as on the roughness of the surface, but it rarely exceeds 300 m. Above the NBL, the air is lightly stratified due to heat loss to space during nighttime. When the sun rises, the atmosphere begins to be heated from below and convective motions quickly overtake wind-shear turbulence. Penetrative convection follows, gradually eroding upward the stratified layer created during the night and replacing it by the *convective boundary layer* (CBL). Depending on how weak this stratification was above the NBL, penetrative convection may be relatively fast, as is the case depicted in Figure 13.1. Wind-induced turbulence is much weaker than convection-induced turbulence, except at the base of the CBL where both have equal intensity. That lower region is called the *surface layer*, and, properly speaking, the CBL is remainder of the convective zone. As time passes, the growing convective region eventually reaches as high as it did on the previous day (assuming perfect repeatability from one day to the next). The suppression of heat input from the surface at sunset halts convection, and what used to be the CBL becomes an unagitated fossil mixed layer, called the *residual layer*. Near ground, wind causes friction and creates a new NBL.

Figure 13.2 displays typical vertical profiles of potential temperature and

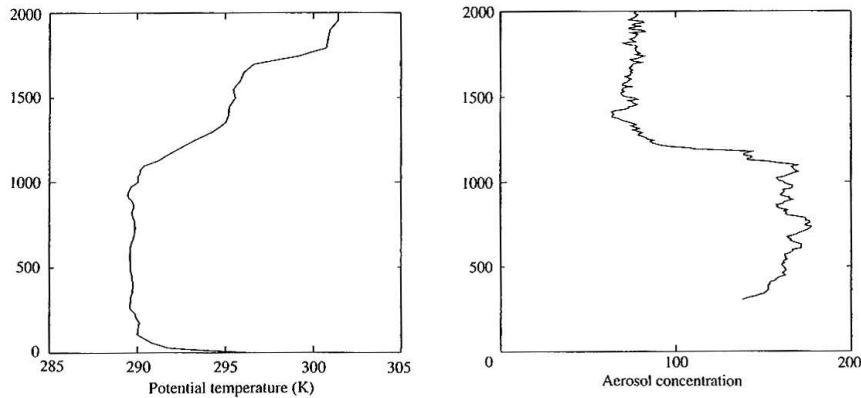


Figure 13.2: Typical vertical profiles in the daytime convective boundary layer. *Left panel:* Potential temperature, exhibiting a thin unstable region in the proximity of the ground, uniformity across the CBL (up to 1000 m) and stratification aloft. *Right panel:* dust particle concentration with high and nearly uniform value (arbitrary units) across the CBL and significantly lower values in the quieter region above, showing that pollution tends to be well mixed across the convective layer but remains almost completely confined to it. [From Garratt, 1992]

pollutant concentration across the CBL and beyond. Potential temperature¹ is the actual temperature corrected for the compressibility effect under altitude-dependent pressure. It starts from a maximum at the ground where heat is delivered and then becomes remarkably uniform across the CBL, indicating a high degree of mixing. The top of the CBL is clearly identifiable by a knee in the profile (just above 1000 m in the example shown), and the potential temperature rises gradually with height, indicative of thermal stratification. Aerosol (dust) concentration, too, reveals a high degree of uniformity across the CBL, with a clearly marked knee in the profile at the same level as the potential-temperature profile. The aerosol concentration above the CBL happens to be quite homogeneous in this example, symptomatic that the thermal stratification succeeded some prior period of mixing, but the fact that the concentration values are markedly different between the CBL and above shows that pollution originates from ground sources, tends to be effectively dispersed across the convective layer but remains almost completely confined to it.

¹See Section 13.3 for a precise definition of this quantity.

13.2 Air Compressibility

Air is a compressible gas, and because pressure drops with altitude as a direct consequence of the hydrostatic balance²

$$\frac{dp}{dz} = -\rho g, \quad (13.1)$$

air aloft is less compressed than air near the ground. The accompanying density difference, due to pressure alone, is not to be confused with possible other differences in density, especially those due to a thermal expansion (decreasing density ρ with increasing temperature T). In other words, thermal stratification needs to be separated from pressure ‘stratification’, the former causing buoyancy forces and the latter not.

Over a height of 1 km, the typical thickness of the CBL, the pressure effect is significant. Indeed, for $\rho = 1.2 \text{ kg/m}^3$, $g = 9.81 \text{ m/s}^2$ and an altitude difference $\Delta z = 1000 \text{ m}$, the (roughly estimated) pressure difference $\Delta p = -\rho g \Delta z$ is about $12,000 \text{ N/m}^2$. Given that the standard atmospheric pressure is $p_a = 101,325 \text{ N/m}^2$, we deduce that pressure drops by about 12% in the first thousand meters of the atmosphere. Under the law of ideal gases, which air obeys quite well despite being a mixture of several gases,

$$p = R\rho T, \quad (13.2)$$

a 12% drop in pressure must correspond to a 12% drop in the product ρT , density times absolute temperature. Thus, either density or temperature or both (both, as we shall see shortly) must decrease with height.

To elucidate the variation of density and temperature with altitude in isolation from thermal stratification, let us imagine a well mixed atmosphere, that is, one composed of air parcels all originating from a common, hypothetical reservoir, all with the same heat content. Then, let them stack up, with some occupying the lower levels under higher pressures and others occupying the higher levels under lower pressures. Those at the bottom will be compressed and will thus have received work from their neighbors on top (like squeezed springs), whereas those on top will be expanded and have performed work onto their neighbors below (like stretched springs). Every particle, therefore, will see its internal energy adjusted according to the elevation it occupies. To this change in internal energy corresponds a change in temperature. Indeed, internal energy is defined in thermodynamics as

$$E = mC_v T, \quad (13.3)$$

where m is the mass of the air parcel, $C_v = 718 \text{ J/(kg}\cdot\text{K)}$ is the heat capacity of air at constant volume (= the amount of heat energy required to raise the temperature of 1 kg of air by one degree Kelvin in a box of constant volume), and T is the absolute temperature (= temperature in degrees Celsius + 273.15).

²We use here the straight d to indicate the vertical derivative because variables discussed in this section depend only on the vertical coordinate.

Internal energy E changes whenever heat is added or subtracted from the air parcel, and work is performed by it or onto it. Conservation of energy demands that the change dE be the added heat dQ ($dQ < 0$ for heat removed) minus the work dW expended by the parcel ($dW < 0$ for work performed onto the parcel):

$$dE = dQ - dW,$$

or, for changes occurring because of a change dz in elevation,

$$\frac{dE}{dz} = \frac{dQ}{dz} - \frac{dW}{dz}. \quad (13.4)$$

In our consideration of a well mixed atmosphere in the absence of thermal effects due to heat fluxes, we take $dQ = 0$. [In the jargon of thermodynamics, we would say that we are considering an *adiabatic change*.] The work done dW is the force times the displacement in the direction of the force but, since we are dealing with a fluid, we use pressure as force per surface (Figure 13.3). Thus,

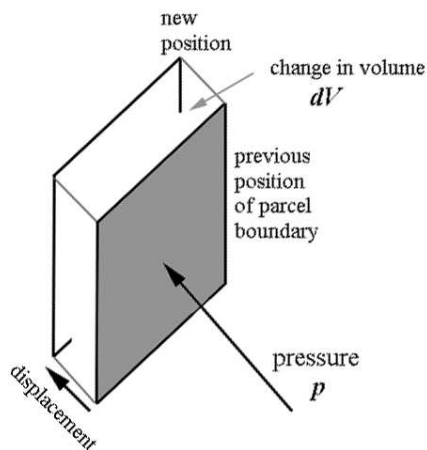


Figure 13.3: How expansion of an air parcel by pressure pushing its boundary outward demands work equal to pressure times change in volume.

$$\begin{aligned} dW &= \text{force} \times \text{displacement} \\ &= \frac{\text{force}}{\text{surface}} \times \text{surface} \times \text{displacement} \\ &= \text{pressure} \times \text{change in volume} \\ &= p dV. \end{aligned}$$

Invoking density as mass per volume ($\rho = m/V \rightarrow V = m/\rho$), we have

$$dW = p d\left(\frac{m}{\rho}\right). \quad (13.5)$$

Putting (13.3) and (13.5) into (13.4) with the additional element that $dQ = 0$, we obtain:

$$\frac{d}{dz}(mC_vT) = -p \frac{d}{dz} \left(\frac{m}{\rho} \right). \quad (13.6)$$

Considering an air parcel of fixed mass m and knowing that the heat capacity C_v is very nearly a constant, we can reduce the preceding equation successively to

$$C_v \frac{dT}{dz} = -p \frac{d}{dz} \left(\frac{1}{\rho} \right), \quad (13.7)$$

and to

$$C_v \frac{dT}{dz} = + \frac{p}{\rho^2} \frac{d\rho}{dz}. \quad (13.8)$$

This equation tells how a change $d\rho$ in density over a change dz in elevation must be accompanied by a change dT in temperature. The + sign indicates that ρ and T vary in the same direction. With increasing altitude, we know that density decreases ($d\rho/dz < 0$) and so must temperature ($dT/dz < 0$).

Taking stock of where we are, we have three fluid properties: pressure p , density ρ and absolute temperature T , linked by the hydrostatic balance (13.1), the equation of state (13.2) and conservation of energy (13.8). This forms a 3-by-3 system of equations. As we are mostly interested in the variation of temperature with height in order to unravel the compressibility effect from differences in heat content, we eliminate p and ρ to obtain a single equation for T . The z -derivative of (13.2) gives

$$\frac{dp}{dz} = R \frac{d\rho}{dz} T + R\rho \frac{dT}{dz}. \quad (13.9)$$

Elimination of dp/dz with (13.1) and of $d\rho/dz$ with (13.8) turns the preceding equation into a single equation for dT/dz :

$$-\rho g = \frac{RC_v\rho^2T}{p} \frac{dT}{dz} + R\rho \frac{dT}{dz}. \quad (13.10)$$

Division by ρ and use of the equation of state (13.2) to eliminate p lead to:

$$(C_v + R) \frac{dT}{dz} = -g.$$

In thermodynamics, the quantity $C_v + R$ is defined as C_p and is called the heat capacity at constant pressure:

$$C_p = C_v + R. \quad (13.11)$$

For air, its value is $C_p = 1005 \text{ J}/(\text{kg}\cdot\text{K})$. With this definition, the vertical temperature gradient is found to be:

$$\frac{dT}{dz} = - \frac{g}{C_p}, \quad (13.12)$$

which, surprisingly, is a constant.

This is the rate at which temperature decreases with elevation in a well mixed atmosphere, called a *neutral atmosphere*, i.e. one in which all air parcels are interchangeable without buoyancy forces. The value of this gradient is

$$\begin{aligned} \Gamma &= \frac{g}{C_p} = \frac{9.81 \text{ m/s}^2}{1005 \text{ J/kg} \cdot \text{K}} \\ &= 0.00976 \text{ K/m} \end{aligned} \quad (13.13)$$

or about 1°C with every 100 m of altitude. This explains why air is colder on a mountain top and snow persists there longer than in the valley below. Over the vertical extent of the atmospheric boundary layer ($h \simeq 1000$ m or more), the temperature decreases from bottom to top by about 10°C or more. The gradient Γ is fundamental in meteorology and is called the *dry adiabatic lapse rate*. The adjective ‘dry’ in this expression refers to the fact that the value of the temperature gradient actually varies with moisture, which was assumed to be nil in the preceding developments. In a moist atmosphere, the lapse rate is slightly less than in a dry atmosphere. The interested reader can find details in Curry and Webster (1999, Section 6.5) and Holton (2004, Section 9.5.2).

13.3 Atmospheric Stability

By definition, the adiabatic lapse rate is the rate at which temperature decreases with height in a well mixed, neutral atmosphere. Thermal stratification thus arises when the vertical temperature variation is other than the adiabatic lapse rate. In the atmosphere, we therefore need to define the stratification N no longer from $N^2 = \alpha g dT/dz$, as was done in Sections 4.2.1 and 5.2, but from

$$N^2 = \alpha g \left(\frac{dT}{dz} + \Gamma \right), \quad (13.14)$$

so that it vanishes in a neutral atmosphere (when $dT/dz = -\Gamma$). We should also note that the thermal expansion coefficient α is not constant when the compressibility effect is significant. The general definition of α is minus the relative rate at which density changes with temperature when pressure is held constant:

$$\alpha = - \left. \frac{1}{\rho} \frac{d\rho}{dT} \right|_{\text{at constant pressure}} \quad (13.15)$$

For an ideal gas, $\rho = p/RT$, this derivative is

$$\frac{\partial \rho}{\partial T} = - \frac{p}{RT^2} = - \frac{\rho}{T},$$

leading to

$$\alpha = \frac{1}{T}. \quad (13.16)$$

Thus, the thermal expansion coefficient of an ideal gas is the inverse of its the absolute temperature. The accurate expression for the stratification frequency N is then

$$N^2 = \frac{g}{T} \left(\frac{dT}{dz} + \Gamma \right). \quad (13.17)$$

The discussion of atmospheric gravitational stability then proceeds as follows (Figures 13.4 and 13.5).

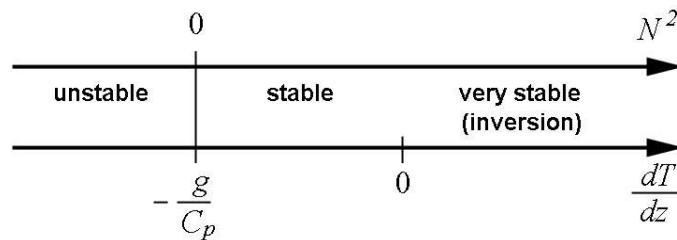


Figure 13.4: Criterion for gravitational stability in the atmosphere, in terms of the quantity N^2 defined in (13.17) and the vertical temperature gradient dT/dz . When $dT/dz > 0$, the atmosphere is very stable and meteorologists call such a situation an *inversion*.

- When $N^2 < 0$, i.e., when the temperature decreases upward faster than at the rate of 1°C per 100 m, the atmosphere is top heavy and convection quickly takes place, bringing the temperature gradient to the neutral level of $-g/C_p$.
- When $N^2 > 0$, i.e., when the temperature decreases with height less fast than 1°C per 100 m or increases with height, the atmosphere is bottom heavy, the air is stratified in a way that satisfies gravity, and no mixing occurs unless a source of mechanical energy exists (ex. a wind shear creating turbulence). The sub-case $dT/dz > 0$ is not particularly significant, but meteorologists like to call it *inversion*, because it is contrary to the common case in which temperature decreases with height. Needless to say, an inversion is a case of strong gravitational stability.

From an environmental air quality perspective, the unstable and neutral cases are the most beneficial because the lack of impeding buoyancy forces

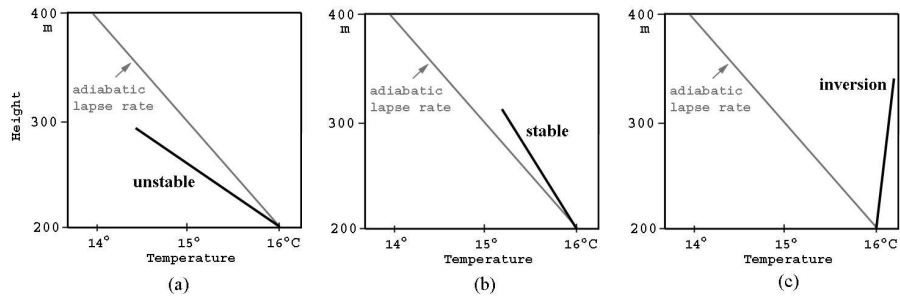


Figure 13.5: Actual temperature profiles (black lines) compared to the adiabatic lapse rate (grey line). In case (a), the temperature decreases with height faster than the adiabatic lapse rate, so that a 16°C parcel starting at 200 m and displaced upward undergoes a temperature decrease by decompression (following the grey line) *lesser* than the ambient decrease (black line). The parcel is *warmer* than its surroundings and subject to a *positive* buoyancy. It continues its upward migration, making the situation unstable. In case (b), the temperature decreases with height but less fast than the adiabatic lapse rate, so that a 16°C parcel starting at 200 m and displaced upward undergoes a temperature decrease by decompression (following the grey line again) *greater* than the ambient decrease (black line). The parcel is *colder* than its surroundings and subject to a *negative* buoyancy force. It falls back down from where it came, and the situation is stable. In case (c), the temperature increases with height, and any upward displacement of a parcel would lead to a lower than ambient temperature and therefore to negative buoyancy. The situation is stable.

helps keep the atmosphere well stirred and pollutants dispersed and diluted. The worst case is that of an inversion near the ground, when the air is unstirred and pollution stagnates. An estimated 4000 people died in London in December 1952 during a 4-day episode of smog caused by a combination of a strong and lasting atmospheric inversion and, at the time, unregulated pollution sources (Boubel et al., 1994). The Los Angeles metropolitan area in California is notorious for its atmospheric inversions (Figure 13.6).



Figure 13.6: Smog over Los Angeles seen as a brown haze covering the basin and making the skyscrapers of downtown Los Angeles barely visible. [Photo ©Tom Politeo]

13.4 Potential Temperature

The potential temperature is an adjusted temperature that discounts the pressure (compressibility) effect. In other words, the potential temperature of an air parcel is not its actual absolute temperature but the absolute temperature that it would have if it were brought adiabatically (= without heat added or subtracted) to a reference pressure, ground-level pressure for example. Referring all air parcels to the same pressure is a way of putting them all ‘on the same footing’, so that any remaining temperature difference after the transformation can only be attributed to differences in heat content.

To calculate potential temperature, we thus need a temperature–pressure relation. This can be extracted from (13.9) in which the z -derivative of density is eliminated via the adiabatic change equation (13.8):

$$\begin{aligned}\frac{dp}{dz} &= C_v \rho \frac{dT}{dz} + R\rho \frac{dT}{dz} \\ &= C_p \rho \frac{dT}{dz} .\end{aligned}$$

Elimination of $\rho = p/RT$ allows to cast this equation with all T 's on one side and all p 's on the other:

$$\frac{C_p}{T} \frac{dT}{dz} = \frac{R}{p} \frac{dp}{dz} , \quad (13.18)$$

which is easily integrated over z :

$$C_p \ln \frac{T}{T_0} = R \ln \frac{p}{p_0} , \quad (13.19)$$

in which T and p are the actual temperature and pressure of the air parcel, and T_0 is the temperature it would take if its pressure were changed to p_0 . If p_0 is the reference pressure used to compare the various air parcels, then T_0 is the potential temperature.

It is the tradition to denote the potential temperature by θ . Solving the preceding equation for $T_0 = \theta$ gives the relation that defines the potential temperature:

$$\theta = T \left(\frac{p_0}{p} \right)^{R/C_p} . \quad (13.20)$$

For air, the exponent R/C_p is equal to 0.286. Recall that in (13.20), T stands for the absolute temperature (= temperature in degrees Celsius + 273.15). For convenience, the reference pressure is taken as the standard atmospheric pressure $p_0 = 1 \text{ atm} = 1 \text{ bar} = 101,325 \text{ N/m}^2$.

Now that potential temperature has been defined, it is noteworthy to express the stratification measure N^2 in terms of it. To do this, we calculate the z -derivative of $\ln \theta = \ln T + (R/C_p)[\ln p_0 - \ln p]$:

$$\frac{1}{\theta} \frac{d\theta}{dz} = \frac{1}{T} \frac{dT}{dz} - \frac{R}{C_p p} \frac{dp}{dz} , \quad (13.21)$$

which by virtue of the hydrostatic balance ($dp/dz = -\rho g$) and the equation of state ($p = R\rho T$) can be turned into:

$$\frac{1}{\theta} \frac{d\theta}{dz} = \frac{1}{T} \left(\frac{dT}{dz} + \frac{g}{C_p} \right) . \quad (13.22)$$

Putting this in expression (13.17) for N^2 , we obtain the compact relation:

$$N^2 = \frac{g}{\theta} \frac{d\theta}{dz} . \quad (13.23)$$

The ensuing discussion is obvious, proving the usefulness of the concept of potential temperature:

- When $d\theta/dz < 0$, i.e., when the potential temperature decreases upward, the atmosphere is top heavy and unstable.
- When $d\theta/dz = 0$, i.e., when the potential temperature is vertically uniform, the atmosphere is neutral.
- When $d\theta/dz > 0$, i.e., when the potential temperature decreases upward, the air is stably stratified and the atmosphere is stable.

This explains why potential temperature tends to be constant across the well mixed across the convective boundary layer (Figure 13.2, left panel).

13.5 The Convective ABL

The ABL is generally convective during daytime, when the sun shines and heats the ground surface, which in turns re-emits the radiation in the form of heating infrared rays, thus heating the lowest atmosphere from below.

13.6 The Stable ABL

The ABL is generally stable at night, when radiative heat loss to space cools the atmosphere primarily from below.

13.7 Top-down and bottom-up diffusion

See Garratt (1992) pages 160–163.

13.8 ABL over Rough Terrain and Topography

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13.9 Nocturnal Jet

(Merge this section with “Stable ABL”?)

13.10 Sea Breeze and Land Breeze

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13.11 Application: Smokestack Plumes

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Problems

13-1. During a sunny late morning, the sun delivers a heat flux of 350 W/m^2 , while the wind creates a ground stress of 0.077 N/m^2 , over a large wheat field. At the time of observation, the height h of the convective boundary layer (CBL) is estimated at 800 m. Determine the following six quantities: the turbulent correlation $\overline{w'T'}$ at the ground, the vertical velocity scale w_* of the rising thermals, the wind friction velocity u_* , the Monin-Obukhov length L (with sign), and the wind speeds at level $z = L$ and at the top of the CBL, $z = h$.

Useful numbers are: Reference air density $\rho_0 = 1.225 \text{ kg/m}^3$, heat capacity of air $C_p = 1005 \text{ J/(kg}\cdot\text{K)}$, thermal expansion coefficient of air $\alpha = 3.47 \times 10^{-3} \text{ K}^{-1}$, and von Kármán constant $\kappa = 0.41$.

13-2.

13-3.

13-4.

13-5.

13-6.

13-7.

13-8.

13-9. By the time the sun rises over the horizon, a smokestack with effective height of 250 m is immersed in an inversion with stratification N equal to 0.02 s^{-1} . The sun provides a heat flux growing linearly in time at the rate of 100 W/m^2 per hour since sunrise. How many minutes or hours after sunrise do you expect fumigation to occur?

13-10.

Chapter 14

NEXT CHAPTER

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THAT CHAPTER 14 CAN BEGIN ON AN ODD PAGE.