Transmission Lines

Introduction

Often you need to transmit a voltage from one physical place to another within a circuit (sometimes over long distances). In lab, we have been using a variety of single-conductor cables for this purpose. In general, however, using these types of cables is not a good solution since these cables are prone to picking up noise (e.g., WDRC, 60 Hz harmonics), as you observed in Lab #2.

A better solution is to use a "shielded" cable such as coaxial, or coax for short, to transmit signals over distances greater than a few centimeters. Using coax cuts down on noise pick-up but it is not perfect. There are two important effects which can cause the output at the end of a long coax cable to be different from the input.

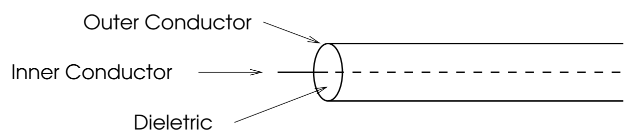


Figure 1: Coaxial cable.

Distributed Capacitance Along the Line

A coaxial cable is made with an inner and outer conductor separated by a dielectric material, as show in Figure 1. Any parallel conductors have a built-in capacitance. The coaxial line has a distributed capacitance that is usually on the order of 100 pF/meter. This value depends on the geometry of the two conductors and the dielectric material (in P41 you may calculate it.) For example, 20 meters of coax looks like a C = 2000 pF between the two conductors. This C, together with the internal resistance of your function generator (typically 50 Ω for reasons explained below) make an RC low-pass filter, shown in Figure 2, with a 3-dB point at $f = 1/2\pi RC \approx 1.6$ MHz.

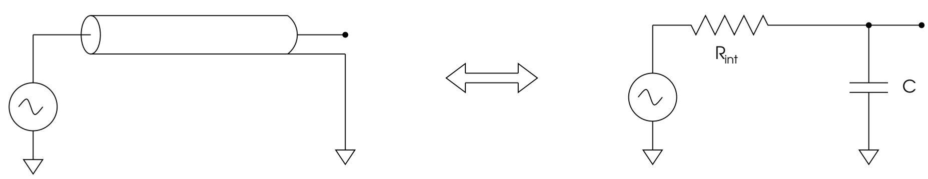


Figure 2: Signal generator and transmission line as a low-pass filter.

Reflections on the Transmission Lines

When the wavelength of the electrical signal you are trying to transmit becomes comparable to the length of the cable your are using transmitting, the signal at either ends of the cable, in general, will not be the same. In this case we must use Maxwell's Equations and we will see that the signals propagate as waves along our coax.

To see this effect we use a better model of the cable which incorporates distributed L as well as C along the cable as shown in Figure 3. Note that there is also some distributed R, which leads to losses, but we ignore it here since it is small for good coax.

Applying the current-voltage characteristics of the distributed L and C along a segment of the line dx:

$$dV = -L\,dx\,\frac{dI}{dt} \qquad (1)$$

$$dI = -C\,dx\,\frac{dV}{dt} \qquad (2)$$

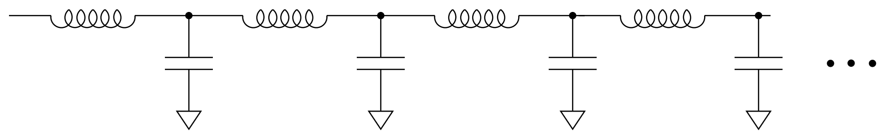


Figure 3: Model of a transmission line as distributed L and C.

where dV is the drop across one of the little inductors in Figure 3, and dI is the current through the paired capacitor. The minus sign in (1) reflects Lenz' Law. Dividing both equations by dx, taking d/dt of (1) and d/dx of (2), and equating the terms $d^2V/dtdx$, we obtain the following "wave equations" for I and V:

$$\frac{d^2I}{dx^2} = LC\frac{d^2I}{dt^2} \qquad \text{and} \qquad \frac{d^2V}{dx^2} = LC\frac{d^2V}{dt^2} \tag{3}$$

where the equation for V is obtained in a similar manner. Solutions for the voltage and current of these equations go as

$$V, I \sim e^{j(kx \pm \omega t)} \tag{4}$$

where $\omega/k = 1/\sqrt{LC}$ is the phase velocity of the waves along the coax line. In words, both V and I propagate along the line as traveling sinusoids at this velocity. If we were to incorporate R along the line, we would have also obtained an exponential decay of the sinusoid amplitude along the line, as we might guess from our study of the LRC circuit. Try plugging (4) into (3) to see that this form of solution works.

Incidentally, the amplitude of V and I have a constant ratio along the line. If you work out the equations, this ratio turns out to be

$$\frac{|V|}{|I|} = \sqrt{\frac{L}{C}} \equiv Z_0 \tag{5}$$

where Z_0 is called the characteristic impedance of the line. Most coaxial cable is designed to have a characteristic impedance of 50 Ω (although 75 Ω varieties are not uncommon). That is why your function generator is also designed to have a "matching" internal resistance of 50 Ω.

If the circuit which you attach to the far end of the line presents an effective impedance (load) of 50 Ω to the line, you are said to have *matched the load to the line.* In this case, the voltage/current wave traveling along the line is perfectly absorbed in the load at the end and no reflected wave occurs. You usually design your circuit for such a matched situation.

If the termination load is not equal to 50 Ω, the voltage/current wave is not all absorbed and is partially reflected. The reflected wave may or may not be inverted, depending on the type of mismatch:

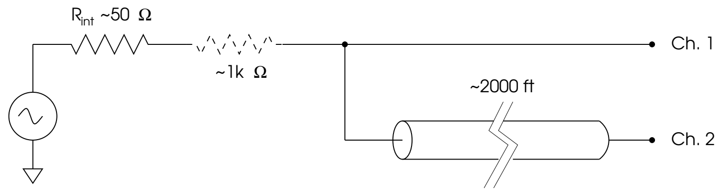
$$Z_{load} = Z_0 \qquad \text{no reflected wave (matched load)} \tag{6}$$
$$Z_{load} > Z_0 \qquad \text{reflected wave and incident wave have same sign} \tag{7}$$
$$Z_{load} < Z_0 \qquad \text{reflected wave is inverted from incident wave} \tag{8}$$

The reflected wave superposes with the incident wave all along the line to cause the voltage all along the line to be a complicated function, generally not equal to what you put in. In particular, for those special frequencies for which the length of the cable equals a multiple of quarter wavelengths, standing waves are created along the line.

Experiment 1: Low-pass Filter

- First measure the capacitance of the long line directly with a meter. There are two available, an expensive modern meter and a capacitance bridge. The latter works similar to the Wheatstone Bridge you built in the first lab. Measure a single 1000 foot spool and both connected as a single 2000 foot cable.

- You can observe directly the low-pass filter effect caused by the internal resistance of the AC voltage source (50 Ω) in combination with the capacitance that you just measured. To see this effect put a sinusoidal signal through the long cable and observe the input and output on your scope (see the figure below).



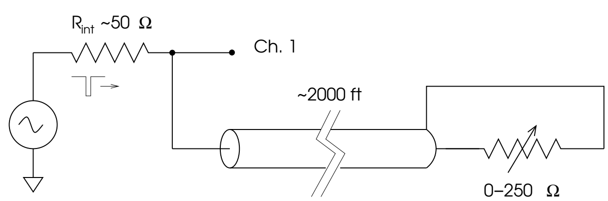
- At low frequencies the output should nearly match the input and at high frequencies the output should fall off with increasing f. Vary the input frequency and determine f_c, the 3-dB point. How does this compare to the expected value?
- use the 1 kΩ resistor in series with the function generator to lower the value of f_c and measure it. How does it compare with the expected value?
- There is also an effective resistance in the coax which leads to loss of signal, even at low frequencies. This loss is called attenuation and it is usually small and can be very small, depending on how much you spend on your coax. A typical number is a few dB per 1000 feet. Measure the amount of attenuation, in dB, for the 2000 feet of coax.

Experiment 2: Standing Waves

- Using the same setup as experiment 1, including the series 1 kΩ, you can observe standing waves frequencies greater than f_c. Measure the frequencies at which the signals are maximum and minimum, noting when the signals are in or out of phase. Use this information to infer the length of the cable. (Hint: Both ends are maxima when an integral number of half-wavelengths exactly fit on the line. You need to know that the phase velocity ($f\lambda$) is less than the speed of light in most coax, a typical number being $0.65c$).
- Terminate the scope end of the long cable with 50 Ω. What happens to the standing wave pattern?
- Short the scope end of the long cable; i.e., terminate with 0 Ω. Doing so produces the condition $Z < Z_0$ at that end of the cable (inverted reflected wave rather than same sign reflected wave). Measure the pattern of minima/maxima versus f again. Are the frequencies of minima/maxima shifted relative to the previous results? Can you explain the shift?

Experiment 3 (time permitting): Traveling Pulse

- Use the function generator as a pseudo pulse generator by setting the output to a square wave of maximum frequency (∼2 MHz) and crank the duty cycle all the way up. The result is a negative 'pulse' with a width of a few microseconds.
- Connect the 2000 foot coax as shown in the figure below. Tune the variable terminator and note how the reflected pulse behaves for various values of Z_{load} relative to the input 50Ω of the signal generator.



- Estimate the length of the cable based on the time-of-flight measurement. How does is compare to the value you obtained from the previous experiment?
- Tune the variable resistor in order to match the input impedance of your function generator. Measure the value of the matched termination with your ohmmeter.
