

Evolutionary Dynamics of Behavior in Social Networks

R. Olfati-Saber

Abstract—In this paper, we use evolutionary dynamics to understand the origin of social norms and dominant behavioral and cultural trends in social networks that lead to emergence of “hubs” such as few companies, politicians, celebrities, and fashion trends with overwhelming popular support. We define the notion of “behavior networks” and introduce a novel social choice model (i.e. mutation model) for evolutionary dynamics of behavior in social networks that exhibits a rich set of emergent phases of evolution. For behavioral networks with Watts-Strogatz small-world topology covering a broad range from highly clustered nearest-neighbor graphs to random graphs, we demonstrate the existence of S-shape phase transition curves relating diversity of behaviors to the mutation rate. This curve suggests the emergence of four key phases of behavior in the society ranging from one or a few dominant behaviors to collapse (or lack of dominance). Preliminary stability analysis and characterization of properties of equilibria are provided that suggest the proposed social choice model is plausible.

Index Terms—evolutionary dynamics, social networks, evolutionary game theory, consensus theory, small-world networks

I. INTRODUCTION

Evolution theory forms the basis of population genetics with applications to biological sciences. Though, the mathematical theory of Darwinian evolution has had a profound impact on social sciences—in particular, economics, game theory, psychology, and linguistics—that led to a formal understanding of the interplay between strategies, learning, and cooperation in agent-based social systems [2], [29], language evolution [25], [18], evolutionary games [17], [9], [19], [1].

In this paper, our main objective is to understand *the origin of social norms and dominant behavioral and cultural trends and emergence of “hubs” such as famous celebrities and highly successful companies in a society*. We use a combination of tools from evolutionary dynamics of populations, complex networks, and control theory to study the evolution of behavior of agents in social networks.

This work is primarily motivated by the desire of the author to develop an analogous behavioral version of flocking and consensus theories [21], [22], [23], [27], [12], [26], [15], [8], [5], [6] for decision-making agents in social networks. The evolutionary theoretic aspect of the work is partially influenced by the evolutionary model of language acquisition in [13], [18] and useful discussions with a few biologists and linguists [14].

We introduce the notion of a *behavior network* that consists of a graph $G = (V, E)$ with a set of nodes representing behaviors b_i and a weight matrix $A = [a_{ij}]$ that represents

the reward a_{ij} of an agent for switching its behavior from behavior b_i to behavior b_j . Several examples of behavior networks in a real-world society are presented in Section II. An agent might change its behavior upon interactions with other agents in a social network. The limited social interactions of the agent restricts the topology G of the behavior network (i.e. meaning that a behavior network is generally not a complete graph).

Let x_i be the fraction of the people in the social network that engage in behavior b_i . Then, $\sum_i x_i = 1$. For example, consider a set of n computer companies that provide laptops. Each has a market-share of x_i of all laptop sales. A customer buying a laptop from company i is engaging in behavior b_i . Upon recommendation from friends—thus, the role of social networks—and advertisements in the media (a key input of social networks), the customer might decide to buy his/her next laptop from company j . The collective behavior of all the agents in the society affects the market-share of the companies in a business cycle. In other words, the x_i 's are reflective of the behavior and beliefs of people in the social network. Similarly, this framework can be applied to various political parties in presidential elections, fashion, and so on.

To study the evolution of x_i 's in behavior networks, we use *replicator-mutator dynamics* [18], [24] (i.e. an evolutionary nonlinear system discussed in Section III). The origin of the replicator-mutator dynamics is the quasi-species model of Eigen & Schuster [7].

The contribution of this paper is to introduce a novel *social choice model* (also known as “learner’s model” in language evolution [16], [13]) that exhibits a rich set of evolutionary phases including emergence of a single dominant behavioral trend (or behavioral flocking), a few dominant behaviors, and lack of social norms (extinction) by changing a scalar mutation rate μ . For small-world behavior networks, we demonstrate the existence of an S-shape phase transition curve for behavior networks with fairly arbitrary connected topologies. This curve shows how the diversity of emergent dominant behaviors change as a function of the mutation rate. Some preliminary stability analysis results are also provided.

Here is an outline of the paper: behavior networks are introduced in Section II. Evolutionary dynamics is discussed in Section III. Our main experimental results including the S-shape phase transition curves are presented in Section IV. Stability results are given in Section IV. Finally, concluding remarks are made in Section VI. .

II. BEHAVIOR NETWORKS

In this section, we discuss a framework for evolutionary dynamics of n species on weighted graphs with interaction

matrix $A = [a_{ij}]$ with non-negative elements a_{ij} . Consider a set of n behaviors $V = \{b_1, \dots, b_n\}$. We refer to a weighted graph $G = (V, E)$ with the set of edges $E = \{(b_i, b_j) : a_{ij} > 0\}$ as a *behavior network* with interaction coefficients A . Each behavior b_i represents the behavior of a group of people in a *social network* (or society) S . Here, the term “behavior” is used in an abstract sense and it refers to features (or aggregates of features) of individual agents in the population.

Let x_i denote the fraction of the population with behavior b_i at time t , assuming that the population of the entire society S is 1, the x_i 's satisfy $\sum_{i=1}^n x_i = 1$. In population genetics, x_i is called the *frequency* of the i th species (i.e. behavior). The interaction coefficient a_{ij} represents the *reward* gained by an agent for conversion from behavior b_j to behavior b_i .

Let us provide some examples that clarify the notion of “behavior” in behavior networks:

a) Companies/Products: In financial markets, each company has a market share x_i of products in a given category and many companies compete with each other to attract more customers. Of course, this translates to increasing their market share. Each product or company is a behavior. A customer who is the user of a laptop from company i is an agent with behavior b_i . This customer might decide to either purchase a laptop from company j upon recommendation of a “friend”, or might end up buying a newer laptop from company i (stay a loyal customer). In the terminology of behavior networks, an agent with behavior b_i might gain a reward for changing its behavior to b_j upon social interactions with other neighboring agents. A common observation is that despite the fact that many companies provide laptops, only very few companies are “dominant behaviors” and own a considerable portion of the market share. The interaction coefficient a_{ij} will later play a role in the rate of transition of the agents from behavior b_i to behavior b_j . A good example of successful products is the iPod by Apple. People of all ages flocked to buy iPods compared to other electronic music players.

b) Political Parties: There are a number of political parties in the US, but the Republicans (R), the Democrats (D), and the Independents (I) form the three dominant parties. The set $V = \{R, D, I\}$ represent the set of behaviors. An agent with behavior D is a Democrat (the reader could guess the rest). An agent with behavior R might gain a reward to convert to behavior D upon interactions with friends and the media (environmental inputs), or vice versa (though, at the moment that is highly unlikely). Similarly, agents with behavior I might change their behavior to R or D by voting for the respective parties. A practical way to determine the interaction coefficients a_{ij} with $i, j \in \{1, 2, 3\}$ is to poll large number of the electorate and then provide an averaged likeliness of an agent with behavior I converting to behavior D and behavior R . The details of the degree of loyalty to the same behavior will be formalized later. Political parties often use celebrities (or “hubs” of social networks) to increase their fraction of the voters. In other words, social networks play a central role in increasing/decreasing the popularity of a behavior. The graph resulting from three dominant parties

is a cycle of length three. Interestingly, the topology of the behavior networks do not share the same features as the underlying structure of their associated social networks (obviously, a cycle does not possess hubs).

c) Grammars: In language acquisition, the grammar spoken by a child is influenced by the child’s parents, teachers, friends, and other individuals who she/he interacts with. Different individuals usually speak different grammars. A child/person speaking grammar i can be viewed as an agent with behavior b_i . Again, an agent might change its behavior upon social interactions with another agent with a different behavior and end up speaking grammar j . In this case, we refer to $G = (V, E)$ as a *grammar network*.

d) Fashion/Social Trends: Anyone familiar with the American culture knows that the popular cultural styles of clothing, hair, music, and dance in 70’s, 80’s, and 90’s are quite different. Each wave of the new social trend made the previous trend obsolete. In case of fashion, most agents abandon the previous behavior and flock to the new behavior in large numbers. Emergence of a single dominant behavior is a form of *behavioral flocking*.

e) Celebrities/TV Shows: A-list celebrities and popular TV shows act as dominant behaviors. There are many actors, actresses, directors, singers, models, and athletes, but there are far less number of A-list celebrities, super models, and superstar athletes and singers. The fans of these celebrities represent the agents and the celebrities themselves in each category are the behaviors.

The fundamental question that we would like to explore is “*What is the origin of emergence of dominant behaviors in social networks?*” We use an evolutionary dynamics framework that heavily relies on the *replicator-mutator dynamics* to investigate the source of the emergence of behavioral trends.

Understanding the underlying mechanism of “how dominant social norms emerge” can shed a light on “how to control the social interactions and the media (inputs to the society) to achieve a desired dominant emergent behavior.”

III. EVOLUTIONARY DYNAMICS

Consider the evolution of a population of n species (behaviors) with a vector of frequencies $x = (x_1, \dots, x_n)$ associated with the vector of behaviors $b = (b_1, \dots, b_n)$ satisfying the normalized condition $\sum_i x_i = 1$. The weight a_{ij} means the *reward* for an agent for conversion from behavior b_j to behavior b_i . Thus, the interaction matrix $A = [a_{ij}]$ can be viewed as the matrix of rewards. We assume $a_{ii} = 1, \forall i$. The fitness of behavior b_i is defined as $f_i = f_0 + \sum_j a_{ij}x_j$ where f_0 is the base fitness (here, we assume $f_0 = 0$). Apparently, the fitness here is frequency (or state) dependent. The *replicator-mutator dynamics* [18], [24] is a nonlinear system in the form

$$\dot{x}_i = \sum_{j=1}^n x_j f_j q_{ji} - \phi x_i, \quad i = 1, \dots, n \quad (1)$$

where $\phi = \sum_i x_i f_i$ is the *average fitness* and $Q = [q_{ij}]$ is the *mutation matrix*. The mutation matrix is a row stochastic

matrix satisfying $\sum_j q_{ij} = 1$. The mutation rate q_{ij} is the rate of conversion from behavior b_i to behavior b_j ($j \neq i$). We refer to a relationship between the mutation matrix Q and the rewards matrix A as a *social choice model*¹.

Define the set of normalized weights $w_{ij} = a_{ij}/(\sum_j a_{ij})$ and let $W = [w_{ij}]$ be the weight matrix of the behavior network $G = (V, E)$. Let L denote the graph Laplacian [23] associated with the graph $G = (V, E)$, i.e. let $D = \text{diag}(d_1, \dots, d_n)$ be the degree matrix of A with $d_i = \sum_j a_{ij}$, then

$$L = I - D^{-1}A = I - W \quad (2)$$

We define Q as the Perron matrix of L [22], i.e. $Q = I - \mu L$, or

$$Q = (1 - \mu)I + \mu D^{-1}A, \quad (3)$$

where $\mu \geq 0$ is the *mutation parameter*. The mutation rates can be explicitly stated as follows:

$$q_{ij} = \mu w_{ij}, \quad q_{ii} = 1 - \mu(1 - w_{ii}).$$

In other words, according to this social choice model, the mutation rates are proportional to relative rewards w_{ij} and the mutation rates are controlled by parameter μ . We refer to this social choice as a first-order model (See Remark 2 for higher-order models).

Remark 1. For the special case with no mutation (or $\mu = 0$), $Q = I$ and system (1) reduces to the *replicator dynamics*

$$\dot{x}_i = x_i(f_i(x) - \phi) \quad (4)$$

that has been extensively studied in game theory and population dynamics literature [9], [24].

Remark 2. More complex models of social choice are possible. In fact, using the first k terms of the expansion of the Perron matrix $Q(\mu) = \exp(-\mu D^{-1}A)$ gives infinite number of social choice models. All these matrices are row stochastic for any arbitrary choice of $\mu > 0$.

Given the matrix of rewards A and a social choice model $Q(\mu)$ and a mutation parameter μ , the evolutionary dynamics of behaviors can be expressed as

$$\dot{x}_i = \sum_{j \in N_i} x_j f_j q_{ji} - \phi x_i, \quad i = 1, \dots, n \quad (5)$$

that only involves interactions between a behavior b_i and its neighboring behaviors $N_i = \{j : a_{ij} > 0\}$.

Remark 3. System (5) is a distributed dynamical system and can be viewed as the generalization of the model studied in [18] with all-to-all interactions.

Let $x(t)$ denote the state of behavioral evolution at time t . A natural question is that how many species exist in steady-state. In population genetics, the effective number of species is known as the *diversity* $n_e(x)$. The diversity is defined as

$$n_e(x) = \frac{1}{\sum_i x_i^2} = 1/\|x\|^2. \quad (6)$$

¹This model in language evolution is called a learner's model.

The diversity varies between 1 and n . The minimum diversity is achieved when a single species has a frequency of $x_{i^*} = 1$ and other species are extinct, $x_j = 0, \forall j \neq i^*$. The surviving species in this case with $n_e = 1$ is called the *dominant behavior*. On the opposite, if all x_i are equal, then $x_i = 1/n, \forall i$ and $n_e = n$ —the diversity achieves its maximum value. In the later case with $n_e = n$, no dominant behaviors exist and almost all agents suffer from extinction for $n \gg 1$.

The above argument on diversity of species motivates us to define three *phases of evolution* for (large) behavior networks:

1) *Behavioral Flocking*: A single dominant behavior emerges, $n_e = 1$.

2) *Cohesion*: A few dominant behaviors emerge, or

$$1 < n_e \ll n.$$

3) *Collapse*: Many dominant behaviors emerge, $1 \ll n_e < n$.

4) *Complete Collapse*: No dominant behaviors emerge and $n_e = n$.

In behavioral flocking, all agents adapt to a single behavior b_{i^*} and essentially that behavior becomes a *social norm*. In the cohesion phase, few dominant behaviors or trends emerge. Finally, in the collapse phase, the society has no dominant behavioral trends.

Let us define a set of equally-distanced angles $\theta_k = 2\pi k/n$ for $k = 1, \dots, n$. Define the *order parameter* of species with frequency x as

$$\rho = \left| \sum_{k=1}^n x_k e^{j\theta_k} \right| \quad (7)$$

where $e^{j\theta_k} = \cos(\theta_k) + j \sin(\theta_k)$ is a unit complex number. Note that $0 \leq \rho \leq 1$. The order parameter frequently appears in measuring the degree of synchronization in oscillator networks. Here, we use it as an additional measure to determine the phase of evolution. In behavioral flocking, $\rho = 1$ whereas in collapse $\rho = 0$. In the cohesion phase of evolution, the order ρ is relatively high.

IV. EXPERIMENTAL RESULTS

Our **main objective** is to explore the role of the structure of behavior networks and the mutation parameter μ on diversity and order of species resulting from the social choice model introduced in (3).

A. Evolution in 1-D Lattice Behavior Networks

A *ring lattice* $C(n, k, a)$ is a cyclic graph with n evenly spaced nodes on a unit circle in which each node is linked to its $2k$ nearest-neighbors with a weight/reward of $a > 0$. Fig. shows the evolution of behaviors with a ring lattice reward graph with $n = 50$ nodes for increasing levels of mutation within the range $0 < \mu \leq 1$.

One concludes that for a ring lattice with $2k/n < 1$, all three phases of evolutionary behavior emerge: 1) small mutation values lead to behavioral flocking with one dominant behavior, 2) slightly higher values lead to cohesive phase, and 3) high values of mutation give rise to collapse

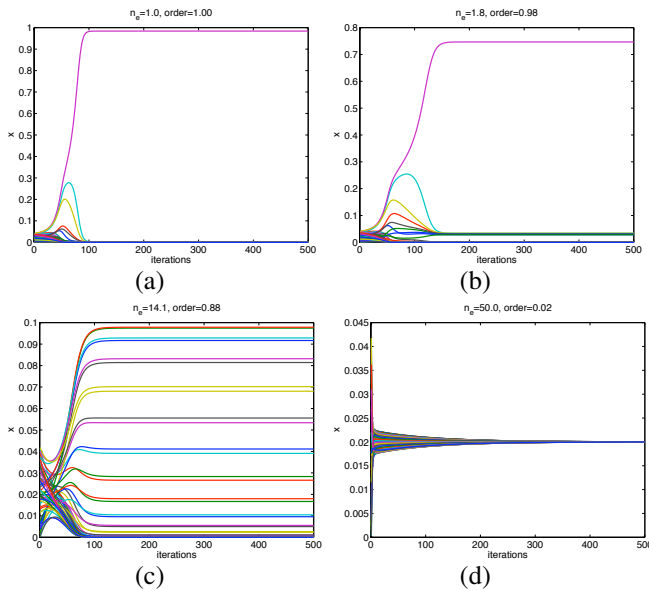


Fig. 1. Evolution of the frequency of behaviors for a ring lattice with $n = 50$, $k = 4$, and $a = 0.5$: (a) behavioral flocking: $\mu = 0.01$, (b) cohesion: $\mu = 0.15$, (c) collapse: $\mu = 1$, and (d) complete collapse: $\mu = 12.5$.

TABLE I
PARAMETERS OF THE EXPERIMENTS IN FIG. 1

Exp.	(a)	(b)	(c)	(d)
mutation μ	0.01	0.15	1	12.5
diversity n_e	1	1.8	14.1	50
order ρ	1	0.98	0.88	0.02

where the diversity is relatively high, and 4) $\mu = n/k$ leads to complete collapse. The diversity can range from $n_e = 1$ to $n_e = n = 50$ and the order can reduce from its maximum $\rho = 1$ to $\rho \approx 0$ for $\mu = 12.5 = n/k$. Let $\bar{\mu} = \min_i 1/(1 - w_{ii})$. The range $\mu \in (0, \bar{\mu}]$ corresponds to *normal mode* of evolution, whereas the seemingly strange interval of $\mu > \bar{\mu}$ corresponds to the *self-destructive mode* of evolution. In the self-destructive mode $q_{ii} < 0$ for at least a node which means the agents migrate out of behavior i .

Fig. 2 shows the *diversity-mutation curve* over the range $[0, 12.5]$ for this ring lattice. Based on our experiments, the shape of this diversity-mutation curve remains invariant for other values of n, k with $2k < n$. In other words, this curve empirically proves that all four phases of evolution emerge by varying the mutation parameter (this is a generic observation).

B. Evolution in Small-World Behavior Networks

Consider a Watts-Strogatz small-world network [28], [20] with $(n, k) = (50, 10)$ and a random rewiring probability of $p = 0.5$. This small-world network can be obtained from a ring lattice by randomly rewiring one end of each of its links with probability of p . The rewards are uniformly set to $a = 0.3$. Fig. 3 shows the diversity-mutation curve for this experiment. Clearly, the diversity-mutation curve is an S-shape phase transition curve that demonstrates the possibility of emergence of all four phases of evolution from behavioral

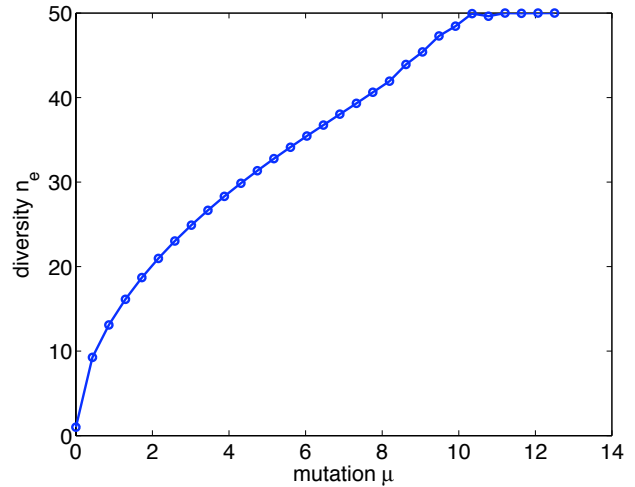


Fig. 2. Diversity-Mutation curve for evolution of behavior on a ring lattice with $(n, k) = (50, 4)$. This empirically proves that all four phases of evolution can emerge depending on μ .

flocking with $n_e = 1$ dominant behavior to (almost) complete collapse with no dominant behaviors. Clearly, the cohesive phase with a few dominant behaviors emerges from the choice of a relatively small mutation parameter $\mu \ll 1$. The slope of the S-shape diversity-mutation curve directly depends on the random rewiring probability p , or density of the short-cuts in the network. Based on our existing observations, the aforementioned features of the S shape curve are generic and do not depend on the size of the network.

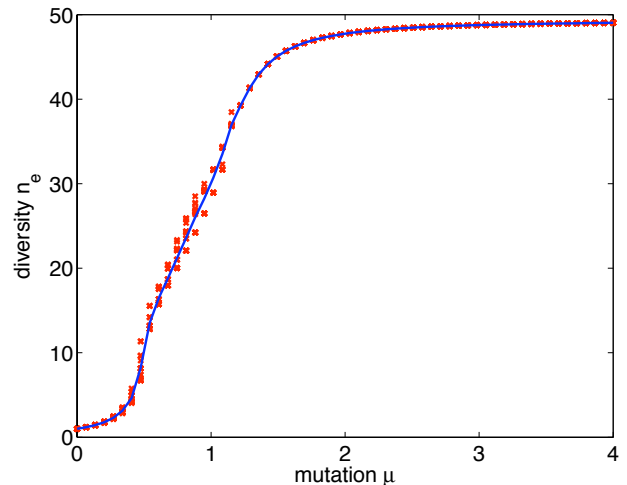


Fig. 3. The S-shape diversity-mutation curve indicating phase transition in evolution of behavior for a Watts-Strogatz small-world network with random rewiring probability of $p = 0.5$ and a uniform reward of $a = 0.3$. The slope of the S-shape curve increases by the percentage of short-cuts p in the small-world network. The S-curve is obtained from averaging over 20 random runs and the actual data is marked by a cross 'x' in red.

C. Evolution in All-to-All Behavior Networks

For complete graphs (or all-to-all networks), the diversity-mutation curve is a sharp S-shape phase transition curve that

either leads to a single dominant behavior or collapse. A complete graph is an ring lattice with $n = 2k + 1$ nodes. Our result is consistent with the special case of complete graphs studied in [18].

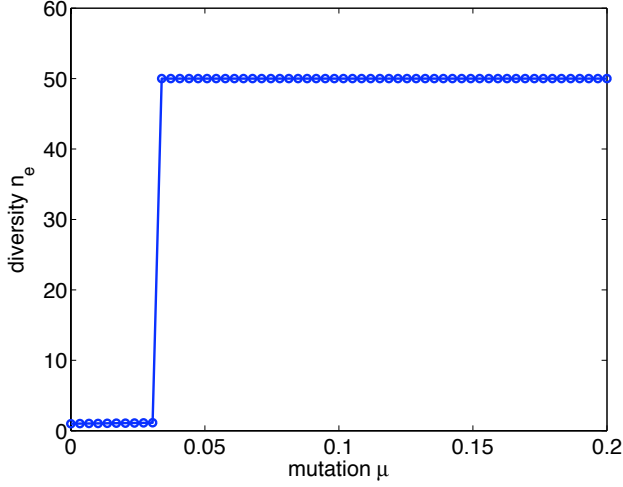


Fig. 4. The sharp S-shape phase transition in the diversity-mutation curve for evolution of behavior in an all-to-all network $n = 50$ nodes.

V. STABILITY ANALYSIS AND EQUILIBRIUM CHARACTERIZATION

In this section, we discuss local stability analysis of the replicator-mutator dynamics and characterize the relationship between the mutation rate and the equilibria/phases corresponding to dominant behaviors.

For the purpose of local stability analysis, the derivation of the linearization of the nonlinear system (1) in explicit analytical form is a challenging task because direct use of the Jacobian linearization method leads to rather cumbersome symbolic calculations and is only feasible for special structured graphs. In the following, we pursue a more tractable approach.

Proposition 1. (linearized evolutionary dynamics) Consider the replicator-mutator dynamics in (1). Let \bar{x} be an equilibrium of the evolutionary dynamics. Then, the linearized evolutionary dynamics in coordinates $z_i = x_i - \bar{x}_i$ for any arbitrary matrix of rewards A is in the form

$$\dot{z}_i = \sum_j \xi_j (q_{ji} - \bar{x}_i) - \bar{\phi} z_i \quad (8)$$

where $\xi_i = \bar{f}_i z_i + \bar{x}_i \phi_i$, $\bar{f}_i = \sum_j a_{ij} \bar{x}_j$, $\phi_i = \sum_j a_{ij} z_j$, and $\bar{\phi} = \sum_i \bar{x}_i \bar{f}_i > 0$.

Proof: Note that $f_i = \sum_j a_{ij} (\bar{x}_j + z_j) = \bar{f}_i + \phi_i$. The average fitness ϕ can be written as

$$\phi = \sum_i (\bar{x}_i + z_i) (\bar{f}_i + \phi_i) = \bar{\phi} + \sum_i \xi_i + \psi$$

where $\bar{\phi} = \sum_i \bar{x}_i \bar{f}_i$ is constant, $\xi_i = \bar{f}_i z_i + \bar{x}_i \phi_i$ is linear in z , and $\psi = \sum_j z_j \phi_j$ is quadratic in z . By direct calculation,

we get

$$\begin{aligned} \dot{z}_i &= \sum_j [(\bar{x}_j + z_j)(\bar{f}_j + \phi_j) q_{ji}] \\ &\quad - [\bar{\phi} + \sum_k \xi_k + \psi] (\bar{x}_i + z_i) \\ &= [\sum_j \bar{x}_j \bar{f}_j q_{ji} - \bar{\phi} \bar{x}_i] + [\sum_j z_j \phi_j q_{ji} - (\psi + \sum_k \xi_k) z_i] \\ &\quad + [\sum_j (\bar{x}_j \phi_j + z_j \bar{f}_j) q_{ji} - \bar{\phi} z_i - \bar{x}_i (\sum_j \xi_j)] \end{aligned}$$

In the above equation, the first term is zero because \bar{x} is an equilibrium of the system. The second term contains all higher-order terms (up to 3rd-order) in z_i 's. The last term contains all the linear terms in $z = (z_1, \dots, z_n)^T$. In other words, we obtain the following linear system

$$\dot{z}_i = \sum_j \xi_j q_{ji} - \bar{\phi} z_i - \sum_j \bar{x}_i \xi_j \quad (9)$$

which is the same as system (8). \square

For the case of symmetric rewards $A = A^T$, the overwhelming observation is that the evolutionary dynamics globally asymptotically converges to an equilibrium $x = \bar{x}$. Though, it might be very challenging to establish global stability analysis for replicator-mutator dynamics, the stability or lack of thereof is not the key issue². The main question of interest is to characterize the relationship between the mutation rate μ and the type of emergent equilibria of the evolutionary dynamics in (1) and their diversity. The following result is crucial in derivation of the relation between μ and phases of evolutionary behavior.

Lemma 1. (balance condition) Consider the replicator-mutator dynamics (1) with the social choice model $Q = (1 - \mu)I + \mu W$. Define the following set of functions

$$\eta_i(x) = \frac{(\phi - f_i) x_i}{x_i f_i (w_{ii} - 1) + \sum_{j \neq i} x_j f_j w_{ji}}. \quad (10)$$

Let \bar{x} be an equilibrium of the evolutionary system. Then, the mutation rate μ and \bar{x} must satisfy the following balance condition

$$\mu = \eta_1(\bar{x}) = \eta_2(\bar{x}) = \dots = \eta_n(\bar{x}) \quad (11)$$

Proof: Let x be an equilibrium of the system satisfying

$$x_i (f_i q_{ii} - \phi) + \sum_{j \neq i} x_j f_j q_{ji} = 0, \quad \forall i.$$

Note that $q_{ji} = \mu w_{ji}, \forall j \neq i$ and $q_{ii} = 1 + \mu(w_{ii} - 1)$. Thus, we obtain

$$x_i (f_i - \phi) + \mu x_i f_i (w_{ii} - 1) + \mu \sum_{j \neq i} x_j f_j w_{ji} = 0$$

and this gives $\mu = \eta_i(x)$. Since, this equality holds for all i , the balance condition in the question must hold for an equilibrium $x = \bar{x}$. \square

²For directed graphs, it is quite well-known that the replicator dynamics (or system (1) with $\mu = 0$) is not stable for a directed cycle of length 3 with equal weights.

Here is our main result on characterization of the mutation level necessary for emergence of various evolutionary phases.

Proposition 2. *Let \bar{x} be an equilibrium of the replicator-mutator dynamics with $Q = (1 - \mu)I + \mu W$. Then, the following statements hold:*

- (i) *An absolutely dominant behavior results from $\mu = 0$ (i.e. replicator dynamics).*
- (ii) *For large behavior networks $n \gg 1$, a single relatively dominant behavior ($x_i = 1 - \epsilon$ and $x_j = O(\epsilon/n)$ for all j) can only emerge from evolution with a relatively small μ .*

Proof: Part (i) can be formally stated as follows: if \bar{x} has a element $\bar{x}_i = 1$ and $\bar{x}_j = 0, \forall j \neq i$ corresponding to behavioral flocking, then $\mu = 0$. Note that $a_{ii} = 1$, thus

$$f_i = a_{ii}x_i + \sum_{j \neq i} a_{ij}x_j = 1 + 0 = 1$$

and $\phi = \sum_j x_j f_j = x_i f_i = 1$. From the balance condition lemma, $\mu = g_i(\bar{x}) = 0$ because $f_i = \phi = 1$.

To show Part (ii), we ignore terms of order $O(\epsilon^2)$ and $O(\epsilon/n)$ as $n \gg 1$. We have

$$f_i = a_{ii}x_i + \sum_{j \neq i} a_{ij}x_j = 1 - \epsilon + \frac{C_i \epsilon}{n} \approx 1 - \epsilon$$

and $f_j \approx a_{ji}\epsilon$. Hence $\phi = \sum_k x_k f_k = (1 - \epsilon)^2 + O(\epsilon^2/n) \approx 1 - 2\epsilon$. From the balance condition lemma, one concludes that

$$\mu = g_i(\bar{x}) \approx \frac{-\epsilon x_i}{(1 - \epsilon)x_i(w_{ii} - 1)} = \frac{\epsilon}{(1 - w_{ii})(1 - \epsilon)}$$

or $\mu = O(\epsilon)$ and thus $\mu \ll 1$ (note that $w_{ii} < 1$). \square

VI. CONCLUSIONS

An evolutionary dynamics approach was presented for the study of the evolution of behavior in social networks. A novel social choice model (i.e. mutation model) was introduced for the replicator-mutator dynamics. It was demonstrated that all four possible phases of evolutionary behavior emerge from the resulting evolutionary dynamics on a ring lattice by changing a scalar mutation rate μ . For behavior networks with Watts-Strogatz topologies, the existence of S-shape diversity-mutation curves were experimentally established that again suggests the emergence of four key behavioral phases. In particular, one concludes that a slow rate of mutation leads to emergence of a few dominant behaviors, or social trends. Further investigations are needed to explore the connections between the emergence of hubs in scale-free networks [3], [4] and network formation [10], [11] and the evolutionary model proposed in this paper.

ACKNOWLEDGMENT

The author would like to thank Jeff Shamma for our discussions on evolutionary game theory and Emilio Frazzoli for his comments on the preliminary version of this work. Many thanks to Charles Taylor and Edward Stabler for their valuable insight into linguistic interpretation of the results.

REFERENCES

- [1] G. Arslan and J. S. Shamma. Anticipatory learning in general evolutionary games. *Proceedings of the 45th IEEE Conference on Decision and Control*, pages 6289–6294, Dec. 2002.
- [2] R. Axelrod. *The Evolution of Cooperation*. Basic Books, New York, 1984.
- [3] A.-L. Barabási and R. Albert. Emergence of scaling in random networks. *Science*, 286:509–512, 1999.
- [4] A.-L. Barabási, R. Albert, and H. Jeong. Mean-field theory for scale-free random networks. *Physica A*, 272:173–197, 1999.
- [5] V. Blondel, J. M. Hendrickx, A. Olshevsky, and J. N. Tsitsiklis. Convergence in multiagent coordination, consensus, and flocking. *44th IEEE Conference on Decision and Control, 2005 and 2005 European Control Conference (CDC-ECC '05)*, pages 2996–3000, Dec. 2005.
- [6] J. Cortés. Distributed algorithms for reaching consensus on arbitrary functions. *Automatica (submitted)*, Oct. 2006.
- [7] J. Eigen and P. Schuster. *The Hypercycle: A Principle of Natural Self-Organization*. Springer-Verlag, 1979.
- [8] Y. Hatano and M. Mesbahi. Agreement over random networks. *IEEE Trans. on Automatic Control*, 50(11):1867–1872, 2005.
- [9] J. Hofbauer and K. Sigmund. *Evolutionary Games and Population Dynamics*. Cambridge University Press, Cambridge, UK, 1998.
- [10] M. O. Jackson and B. W. Rogers. Search and the formation of large networks: when and why do we see power laws and small words? *preprint*, pages 1–35, May 2004.
- [11] M. O. Jackson and B. W. Rogers. The economics of small-worlds. *Journal of European Economic Association*, 3(2–3):617–627, April/May 2005.
- [12] A. Jadbabaie, J. Lin, and A. S. Morse. Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Trans. on Automatic Control*, 48(6):988–1001, June 2003.
- [13] N. L. Komarova, P. Niyogi, and M. A. Nowak. The evolutionary dynamics of grammar acquisition. *Journal of Theoretical Biology*, 209:43–59, 2001.
- [14] Y. Lee, C. E. Taylor, and E. P. Stabler. Personal Communication, 2004.
- [15] L. Moreau. Stability of multiagent systems with time-dependent communication links. *IEEE Trans. on Automatic Control*, 50(2):169–182, 2005.
- [16] P. Niyogi and R. C. Berwick. Evolutionary consequences of language learning. *Linguist. Philos.*, 20:697–719, 1997.
- [17] M. A. Nowak, S. Bonhoeffer, and R. M. May. Spatial games and the maintenance of cooperation. *Proc. of National Academy of Sciences*, 91:4877–4881, 1994.
- [18] M. A. Nowak, N. L. Komarova, and P. Niyogi. Evolution of universal grammar. *Science*, 291:114–118, 2001.
- [19] M. A. Nowak and K. Sigmund. Spatial games and the maintenance of cooperation. *Science*, 303(5659):793–799, 2004.
- [20] R. Olfati-Saber. Ultrafast consensus in small-world networks. *Proceedings of the 2005 American Control Conference*, pages 2371–2378, June 2005.
- [21] R. Olfati-Saber. Flocking for Multi-Agent Dynamic Systems: Algorithms and Theory. *IEEE Trans. on Automatic Control*, 51(3):401–420, Mar. 2006.
- [22] R. Olfati-Saber, J. A. Fax, and R. M. Murray. Consensus and cooperation in networked multi-agent systems. *Proceedings of the IEEE*, 95, Jan. 2007.
- [23] R. Olfati-Saber and R. M. Murray. Consensus problems in networks of agents with switching topology and time-delays. *IEEE Trans. on Automatic Control*, 49(9):1520–1533, Sep. 2004.
- [24] K. M. Page and M. A. Nowak. Unifying evolutionary dynamics. *Journal of Theoretical Biology*, 219(5659):93–98, 2002.
- [25] S. Pinker and P. Bloom. Natural Language and Natural Selection. *Brain and Behavioral Sciences*, 13:707–784, 1990.
- [26] W. Ren and R. W. Beard. Consensus seeking in multiagent systems under dynamically changing interaction topologies. *IEEE Trans. on Automatic Control*, 50(5):655–661, 2005.
- [27] J. N. Tsitsiklis, D. P. Bertsekas, and M. Athans. Distributed asynchronous deterministic and stochastic gradient optimization algorithms. *IEEE Trans. on Automatic Control*, 31(9):803–812, Sep. 1986.
- [28] D. J. Watts and S. H. Strogatz. Collective dynamics of ‘small-world’ networks. *Nature*, 393:440–442, June 1998.
- [29] H. P. Young. *Individual strategy and social structure: an evolutionary theory of institutions*. Princeton University Press, 2001.