Flocking with Obstacle Avoidance: Cooperation with Limited Communication in Mobile Networks

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Abstract

In this paper, we provide a dynamic graph theoretic framework for flocking in presence of multiple obstacles. In particular, we give formal definitions of nets and flocks as spatially induced graphs and define flocking. We introduce the notion of framenets and describe a procedure for automatic construction of an energy function for groups of agents. The task of flocking is achieved via dissipation of this energy according to a protocol that only requires the use of local information. We show that all three rules of Reynolds are hidden in this single protocol. Three types of agents called α, β, and γ agents are used to create flocking. Simulation results are provided that demonstrate flocking by 100 dynamic agents.

1 Introduction

A special behavior of large number of interacting dynamic agents called "flocking" has attracted many researchers from diverse fields of scientific and engineering disciplines. The term "flocking" in English means "moving together in large numbers". This behavior exists in the nature in the form of flocking of birds, schooling of fish, and swarming of bacteria.

Reynolds introduced three *ad-hoc* protocols for autonomous agents moving in a 3-D space called "boids" [5]. The combination of these three protocols led to creation of the first animation of flocking in 1987. In [5], no equations are provided that specify the flocking rules for boids. A simulation of the alignment rule of Reynolds with explicit statement of the equations of the alignment rule is given in Vicsek *et al.* [6]. A similar attitude alignment problem was recently investigated by Jadbabaie *et al.* [2] with a convergence analysis for asymptotic alignment. To the best of our knowledge, no analysis of the other two rules with a motion planning nature has been presented yet. One of the contributions of this paper is the derivation and analysis of a unified and advanced form of Reynolds rules.

In this paper, our main goal is design and analysis of distributed algorithms for large number of dynamic agents that enables them as a group to perform coordinated tasks. The primary tasks of interest are *split, rejoin, and squeezing maneuvers* that are performed as by-products of flocking with obstacle avoidance. We present a dynamic graph theoretic framework that provides the necessary tools for design and analysis of distributed controllers for flocks of dynamic agents. This framework is the result of combination of ideas from various fields including dynamical systems, graph theory, mechanics, distributed computation, and complex systems. We introduce three types of agents called α, β, and γ agents with specific tasks that enable a group of agents to perform flocking in presence of multiple obstacles. These obstacles break the communication/sensing links that cause *switching of the topology in mobile networks* [4]. The stability analysis for such switching systems is rather challenging.

An outline of the paper is as follows: in Section 2 nets and flocks are defined as spatially induced graphs. In Section 3, definitions of framenets and flocking are given and construction of the energy of flocks is explained. The rules of flocking and some of the main results are given in Section 4. Obstacle avoidance and simulation results are presented in Section 5. Finally, in Section 6, concluding remarks are made.

2 Nets: Spatially Induced Graphs

In this section, we define several basic notions in graph theory and introduce the notion of a flock as a spatially induced graph. A graph is denoted by $G = (\mathcal{V}, \mathcal{E})$ with $\mathcal{V}$ as the set of nodes and $\mathcal{E}$ as the set of edges of the graph. The order of a graph is the number of the nodes of the graph $n = |\mathcal{V}|$. An edge is denoted by ij, (i, j) or (v_i, v_j) with $i, j \in \mathbb{N}$ as the node indices and $v_i, v_j \in \mathcal{V}$.

Consider a set of *dynamic agents* (or nodes) with the point-mass dynamics

$$\left\{ \begin{array}{lcl} \dot{q}_i & = & p_i, \\ \dot{p}_i & = & u_i, \end{array} \right. \qquad (1)$$

where $q_i, p_i, u_i \in \mathbb{R}^d$ (e.g. $d = 2, 3$) denote the position, velocity, and control input of the ith agent, respectively. Denote the configuration of all nodes by $q = \mathrm{col}(q_i) \in \mathbb{R}^{nd}$ with $n = |\mathcal{V}|$. A *spherical neighborhood* of a node v_i is a closed ball $B(q_i, r_i)$ of radius r_i around $x = q_i$ in $\mathbb{R}^d$. Let $\mathbf{r} = \mathrm{col}(r_i)$ be the vector of radii of all nodes. Given a fixed set of radii $\mathbf{r}$, any configuration q induces a graph $G(q) = (\mathcal{V}, \mathcal{E}(q))$ called a *net* that is determined by a *spatial adjacency matrix* $\mathcal{A}(q) = [a_{ij}(q)]$ with elements

$$a_{ij}(q) = \left\{ \begin{array}{ll} \rho(\|q_j - q_i/r_i), & j \neq i, \\ 0, & i = j, \end{array} \right. \qquad (2)$$

where $\rho : \mathbb{R}_{\geq 0} \to [0, 1]$ is called the *influence map*. Apparently, a net is a *spatially induced graph*.

An influence map satisfies two properties: i) $\rho(0) = 1$ and ii) $\rho(z) = 0$ for all $z > 1$. We refer to $a_{ij}(q)$ as the *influence of node i on node j*. Each node has zero influence on itself. The set of *neighbors* of node i is defined as

$$N_i = N_i(q) = \{j : a_{ij}(q) > 0\}, \qquad (3)$$

i.e. the set of nodes that are positively influenced by node i.

Let us consider the example of a *discontinuous influence map* $\rho(z)$ with a jump at $z = 1$ that takes the value 1 over the interval $[0, 1]$ and zero elsewhere. Assume $r_i = r$ for all nodes. This gives a symmetric adjacency matrix $\mathcal{A}(q)$ with $0, 1$ elements defined by

$$a_{ij}(q) = \left\{ \begin{array}{ll} 1, & q_j \in B(q_i, r), j \neq i \\ 0 & \text{otherwise.} \end{array} \right. \qquad (4)$$

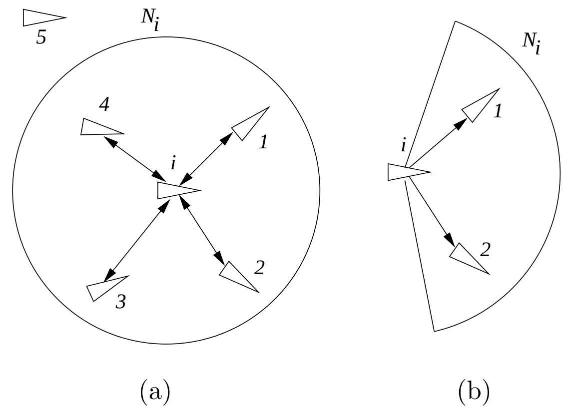


Figure 1: (a) A spherical neighborhood (or shell) and (b) a conic neighborhood.

Figure 1(a) shows an example of a node with a spherical neighborhood and its set of neighbors N_i. As a result, the net $G(q)$ is undirected (i.e. $a_{ij}(q) = a_{ji}(q)$ for all $i \neq j$). Notice that for two nodes i and j with $r_i < r_j$ and $r_i < \|q_j - q_i\| < r_j$, $a_{ij} = 0$ but $a_{ji} = 1$. Thus, nodes with *heterogeneous neighborhoods* give rise to *directed nets*. Furthermore, the configuration of agents with position, attitude, and homogeneous conic neighborhoods as shown in Figure 1(b) could induce directed nets (See [3] for more details). The use of conic neighborhoods for flocking is due to Reynolds [5].

Now, let us consider the example of a C^1-*smooth influence map* $\rho_\delta(z)$ defined by

$$\rho_\delta(z) = \left\{ \begin{array}{ll} 1, & z \in [0, \delta] \\ \frac{1}{2}[1 + \cos(\pi \frac{(z-\delta)}{(1-\delta)})], & z \in [\delta, 1] \\ 0 & \text{otherwise.} \end{array} \right. \qquad (5)$$

where the parameter $\delta \in (0, 1)$ determines the size of the region of a spatial neighborhood with the maximum influence. Clearly, $\rho'_\delta(z)$ is a continuous function that vanishes over $[0, \delta] \cup [1, \infty)$ and is negative over the transition interval $(\delta, 1)$ where the influence function reduces from 1 to zero. The derivative of the influence function is uniformly bounded, i.e. $-\frac{\pi}{2(1-\delta)} \leq \rho'(z) \leq 0$ for all $z \geq 0$ (e.g. $|\rho'_\delta(z)| < 5.25, \forall z$ with $\delta = 0.7$).

From the preceding examples of discontinuous and smooth influence maps, it is apparent that the regularity properties of the influence map determines the regularity of the adjacency matrix of nets. On the other hand, the choice of (homogeneous) spherical or conic neighborhoods specifies whether a net is an undirected or directed graph. We are ready to formally define a flock for the general case of a directed net.

Definition 1. (flock) A *flock* is a weakly connected directed net, or a connected undirected net.

We remind the reader that a digraph is called weakly connected if there exists a path that connects any two distinct nodes of the graph irrespective of the direction of the edges that constitute the path. Throughout this paper, we assume all the nets are undirected and induced by spherical neighborhoods with identical radii, i.e. $r_i = r$ for all i.

The notion of spatially induced graphs, or nets, allow one to define *spatial Laplacian matrices* in the form

$$L(q) = \Delta(q) - \mathcal{A}(q) \tag{6}$$

that depend on the state of the dynamic agents where $\Delta(q)$ is the spatial degree matrix of the net $G(q)$, i.e. $\Delta(q) = [\Delta_{ij}]$ is a diagonal matrix with diagonal elements $\Delta_{ii} = \sum_j a_{ij}(q)$ for $i = 1, \dots, n$.

3 Framenets and Energy of Dynamic Nets

Let us refer to all the nodes of a net/flock as α-*agents*. The *task of an α-agent* at q_i is to maintain a distance d_α (satisfying $0 < d_\alpha < r$) with any neighboring α-agent in a closed ball $B(q_i, r)$. We refer to a net that consists of α-agents as an α-net. The mathematical translation of the task of an α-agent can be expressed in the form

$$\|q_j - q_i\| = d_\alpha, \forall j \in N_i. \tag{7}$$

The specification of (7) is a triplet (n, d_α, r) called the *structural α-net* (or *s_α-net* for brevity) where n is the number of nodes. Consider a set of n points in $\mathbb{R}^d$ with configuration $q \in \mathbb{R}^{nd}$ satisfying the set of algebraic distance-based constraints in (7). Then, q is called a *realization* of the s_α-net (n, d_α, r). In the literature of graph rigidity, the pair (G, q) that consists of a graph $G = (\mathcal{V}, \mathcal{E})$ and a configuration q is called a *framework*. Similarly, a *framenet* is a pair $(G(q), q)$ where $G(q) = (\mathcal{V}, \mathcal{E}(q))$ is a net induced by the configuration q of the nodes. A framenet $(G(q), q)$ is called a *realization* of the s_α-net (n, d_α, r) if $G(q)$ is a net of order n and q is a realization of (n, d_α, r). A framenet $(G(q), q)$ with a weakly connected net $G(q)$ is called a *flock*. Thus, a flock as a net (or graph) is denoted by $G(q)$ and as a framenet is denoted by $(G(q), q)$. Figure 2 gives six framenets as different realizations of a s_α-net (n, d_α, r) with $n = 4$ and $d_\alpha < r < \sqrt{2} d_\alpha$. Notice that the framenets in Figures 2(c) through (f) are flocks.

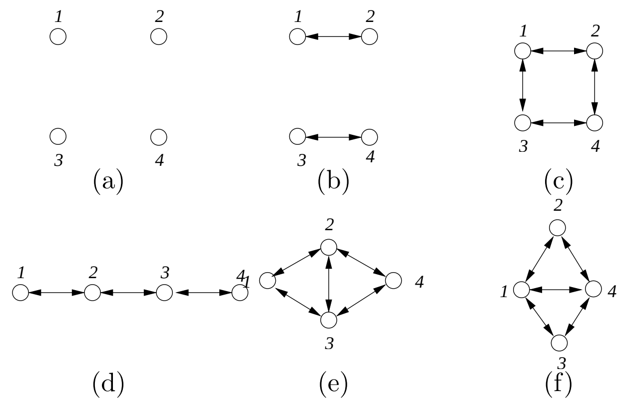


Figure 2: Six realizations of a s_α-net (n, d_α, r) with $n = 4$ nodes.

Remark 1. An intuitive (or visual) way to view framenets is to think of a framenet as a "molecule" and the nodes as the "atoms". The nodes of a framenet establish "bonds" (or edges) if they come within a close proximity of each other. An existing bond in a framenet "breaks apart" (or an edge disappears) if two nodes that are bonded together move more than a distance r apart. Furthermore, only framenets with n nodes that the length of all of their edges is equal to d_α are realizations of a s_α-net (n, d_α, r).

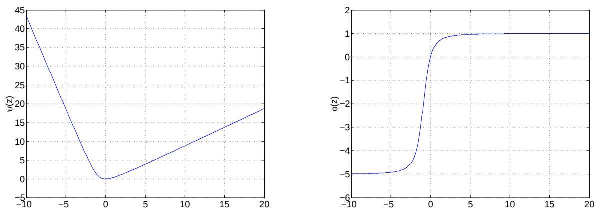
To quantify the degree in which a configuration q is close to a realization of a structural α-net, we define a non-negative and smooth *potential function* $V(q) : \mathbb{R}^{nd} \to \mathbb{R}_{\geq 0}$ in the form

$$V(q) = \sum_{i,j} a_{ij}(q) \psi_\alpha(\eta_{ij}) \tag{8}$$

where $\eta_{ij} = \|q_j - q_i\| - d_\alpha$ is the *edge-deviation* variable. The function $\psi_\alpha(z) : \mathbb{R} \to \mathbb{R}_{\geq 0}$ has a unique global minimum at $z = 0$ and is given by

$$\psi_\alpha(z) = \frac{a+b}{2}(\sqrt{1+(z+c)^2} - \sqrt{1+c^2}) + (\frac{a-b}{2})z,$$

with $b > a > 0$ and $c = |a - b|/2\sqrt{ab} > 0$ (see Figure 3). The choice of $\psi_\alpha(z)$ is such that in a gradient system associated with $V(q)$, it acts as an *attractive/repelling potential*.



(a) (b)

Figure 3: The potential and action functions with parameters $a = 1$ and $b = 5$: (a) $\psi_\alpha(z)$ and (b) $d\psi_\alpha(z)/dz$.

Example 1. Let us consider a framenet with $n = 2$ nodes. The potential function associated with this framenet is given by

$$V(q) = 2\rho_\delta(\frac{\|q_2 - q_1\|}{r})\psi_\alpha(\|q_2 - q_1\| - d_\alpha)$$

and is shown in Figure 4 as a function of the edge-deviation variable $\eta = \|q_2 - q_1\| - d_\alpha$.

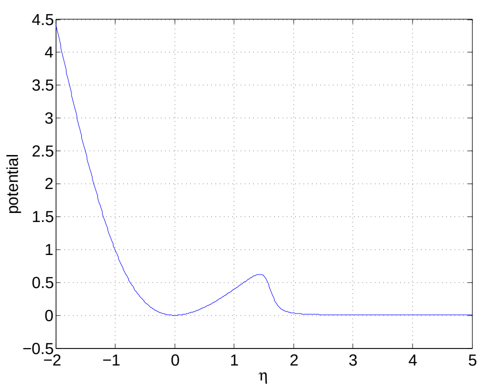


Figure 4: The pair-wise smooth potential function between two α-agents.

The key property of the potential function $V(q)$ is that

$$V(q) = 0 \iff q \text{ is a realization of } s_\alpha\text{-net}.$$

This motivates one to reduce the potential of a framenet $(G(q), q)$ to converge to a framenet that satisfies the set of constraints in (7). However, α-agents are not kinematic agents and their dynamics is $\ddot{q}_i = u_i$. Thus, one has to define an appropriate energy (or Hamiltonian) associated with a net $G(q)$ and attempt to reduce this energy to achieve the objectives of the α-agents (i.e. flocking). Roughly speaking, "flocking" is the process of converging to a *low-energy conformation* of a structural α-net. We formalize this idea in the sequel.

Let $\bar{\cdot}$ denote the average operation on n vectors, i.e. $\bar{q} = \text{Ave}(q) = \frac{1}{n}\sum_{i=1}^n q_i$. One can express the *translational group dynamics* as

$$\text{TGD} : \begin{cases} \dot{\bar{q}} &= \bar{p}, \\ \dot{\bar{p}} &= \bar{u}, \end{cases} \tag{9}$$

Let $\bar{x} = Ave(x)$ and define the $\tilde{\cdot}$ operation as $\tilde{x}_i = x_i - \bar{x}$ and call $\tilde{x}_i$ the *relative* value of x_i. The *relative group dynamics* can be written as

$$\text{RGD} : \begin{cases} \dot{\tilde{q}} &= \tilde{p}, \\ \dot{\tilde{p}} &= \tilde{u}. \end{cases} \tag{10}$$

A *dynamic α-net* is a one-parameter family of graphs $G(q(t))$ embedded into $\mathbb{R}^d$ such that each node is a dynamic system $\dot{x}_i = f(x_i, u_i)$ and the potential function associated with $G(q(t))$ is in the form (8) that is determined by the task of α-agents. Here, the state of each node is $x_i = \text{col}(q_i, p_i)$ and the map f is such that each node is a double-integrator. The graph $G = G(q(t_0))$ is called the *topology* of the dynamic α-net at time $t = t_0$. We define the *structural energy* of a dynamic α-net as follows:

$$H_s(q, p) = V(q) + K_r(\tilde{p}) \tag{11}$$

where K_r is the *relative kinetic energy* given by

$$K_r(\tilde{p}) = \frac{1}{2}\sum_{i=1}^n \|\tilde{p}_i\|^2. \tag{12}$$

The consequences of lowering the structural energy of an α-net to zero are summarized in the following result:

Proposition 1. *(zero structural energy) Let $H_s(q, p)$ be the structural energy of a dynamic α-net. Assume $H_s(q(t), p(t)) = 0$ for $t \in [t_0, \infty)$. Then, for all $t \geq t_0$ the following statements hold: i) $q(t)$ is a realization of the s_α-net (n, d_α, r), ii) all α-agents move with the same velocity (or towards the same direction), iii) the topology of the dynamic α-net remains invariant, and iv) if the α-net $G(q(t_0))$ is a flock, it remains a flock.*

Proof. For the proof of the first three statements see Proposition 2 in [3]. Part iv) follows directly from part iii). □

The *translational energy* of an dynamic α-net can be similarly defined as

$$H_{tr}(q,p) = V_{tr}(\bar{q}) + \frac{1}{2}\|\bar{p}\|^2 \qquad (13)$$

where $V_{tr}(\bar{q})$ is the *translational potential* associated with a (sink/source) point or set (in the sense of stabilization to a set). We have the following *separation principle*:

Proposition 2. *Let $H_s(q,p)$ and $H_{tr}(q,p)$ denote the structural and translational energies of a dynamic α-net with dynamic agents that have double-integrator models. Then, $\dot{H}_s$ does not depend on $\bar{u}$ and $\dot{H}_{tr}$ does not depend on $\tilde{u}$.*

Proof. By direct calculation, we have $V(q) = V(\tilde{q})$ and

$$\dot{H}_s = \langle \nabla V(\tilde{q}), \tilde{p}\rangle + \langle \tilde{p}, \tilde{u}\rangle,$$

which is independent of the choice of $\bar{u}$. Similarly, $\dot{H}_{tr}$ only depends on $\bar{u} = \mathrm{Ave}(u)$. □

The properties of the structural energy of an α-net motivates us to define "flocking" as the following:

Definition 2. (asymptotic flocking) Given the state feedback $u = k(q,p)$, we say a *dynamic α-net* with structural energy $H_s(q,p)$ is *asymptotically structurally stable* if both of the following conditions hold: i) For any $C_0 > 0$ there exists a $C_1 > 0$ such that $H_s(q(0),p(0)) \le C_0 \implies H_s(q(t),p(t)) \le C_1$ for all $t > 0$ and ii) $\lim_{t\to\infty} H_s(q(t),\tilde{p}(t)) = 0$. Moreover, $u = k(q,p)$ achieves *asymptotic flocking* if in addition to conditions i) and ii), the following condition holds: iii) For any $C_0 > 0$ and any initial edge-deviation and velocity satisfying $H_s(q(0),p(0)) < C_0$, there exists a $T > 0$ such that for all $t > T$, $G(q(t))$ is a flock.

Remark 2. Whenever condition ii) of flocking is replaced by $\lim_{t\to\infty} H_s(q(t),\tilde{p}(t)) \le \epsilon$ with $0 < \epsilon \ll 1$, we refer to the corresponding notion of stability as *asymptotic ϵ-flocking* or *practical flocking.*

Next, we introduce our flocking protocol for the interactions among the α-agents.

4 Rule of Flocking: The (α,α) Protocol

In this section, we discuss flocking in lack of any obstacles. The main contribution of this paper is to introduce the notion of *α-nets* and propose *flocking via dissipation of structural energy* of a dynamic α-net.

Consider the following distributed state feedback called the (α,α) *protocol*:

$$u_i^{(\alpha,\alpha)} = \sum_{j\in N_i} \phi_\alpha(\|q_j - q_i\| - d_\alpha)\mathbf{n}_{ij} + c_d(p_j - p_i) \qquad (14)$$

where $c_d > 0$ is the damping coefficient and $\phi_\alpha(z)$ is defined as the following

$$\phi_\alpha(z) = \frac{d\hat{\psi}_\alpha(z)}{dz},\ \hat{\psi}_\alpha(z) = \rho(\frac{z+d_\alpha}{r})\psi_\alpha(z). \qquad (15)$$

The (α,α) protocol provides the interaction rule among α-agents in lack of obstacles. This protocol is *momentum preserving*, i.e. $\sum_i u_i^{(\alpha,\alpha)} = 0$ and $\bar{p} = \frac{1}{n}(\sum_i p_i)$ is an invariant quantity given $u = u^{(\alpha,\alpha)}$. Defining the *stress weights* $s_{ij}(q)$ as

$$s_{ij}(q) = \frac{\phi_\alpha(\|q_j - q_i\| - d_\alpha)}{\|q_j - q_i\|},\ q_j \neq q_i \qquad (16)$$

for $j \in N_i$ (and zero, otherwise), the control input of the ith α-agent can be expressed as

$$u_i^{(\alpha,\alpha)} = \sum_{j\in N_i} s_{ij}(q)(q_j - q_i) + c_d(p_j - p_i) \qquad (17)$$

Thus, the input of each α-agent is the weighted sum of relative positions and velocities of the neighbors of an α-agent w.r.t. that agent. Assuming $c_d = 0$ and $S_i(q) = \sum_{j\in N_i} s_{ij}(q) \neq 0$, we obtain the following rule

$$u_i^{(\alpha,\alpha)} = \sum_{j\in N_i} s_{ij}(q)(q_j - q_i) + c_d(p_j - p_i) \qquad (18)$$

with $q_i^{ave} = (\sum_{j\in N_i} s_{ij}(q)q_j)/S_i(q)$. If equation (18) is interpreted in English it can be stated as "each agent moves towards/away from the weighted average position of its neighbors depending on the sign of $S_i(q)$". The sentence in quotes combines two out of the three flocking rules of Reynolds, i.e. "Flock Centering" and "Collision Avoidance" [5]. For a detailed discussion of the "Alignment" rule and the case $S_i(q) = 0$, please see [3]. Here is the main result of the paper:

Proposition 3. *(flocking via energy dissipation) Given the* (α, α) *protocol in* (14)*, the structural energy* $H_s(q(t), p(t))$ *of the dynamic* α*-net is monotonically decreasing, i.e.* $\dot{H}_s \leq 0$*. Moreover, the topology of the net* $G(q(t))$ *is asymptotically invariant. Furthermore, if* $G(q(t))$ *is asymptotically a flock, then all the* α*-agents asymptotically move with the same velocity.*

Proof. See the proof of Proposition 5 in [3]. □

A framenet $(G(q), q)$ is called *stress-free* if $s_{ij}(q) = 0$ for all i, j. Otherwise, it is called *stressed*. Given the (α, α) protocol, the limiting framenet with an invariant topology might or might not be stress-free. The following result characterizes the set of stress-free framenets.

Proposition 4. *A dynamic* α*-net is stress-free at time* t *if and only if the framenet* $(G(q(t)), q(t))$ *is a realization of the structural* α*-net* (n, d_α, r)*.*

Proof. By direct calculation. □

Figure 5 shows examples of a stressed framenet with $s_{23} > 0, s_{26} > 0$ and a stress-free framenet associated with a structural α-net satisfying $r = kd_\alpha$ with $\sqrt{2} < k < \sqrt{3}$.

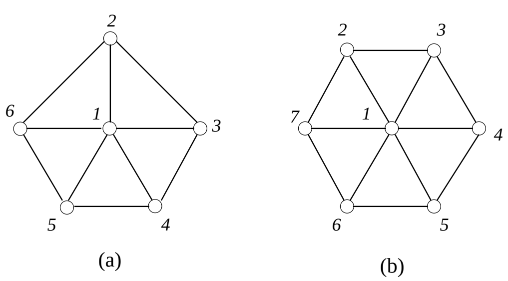


Figure 5: framenets of a s_α-net: (a) a stressed framenet and (b) a stress-free framenet.

Stressed framenets with relatively small stress weights can be viewed as *quasi-realizations* of structural α-nets. To quantify the degree in which a framenet is defected, we define the *defect factor* $\mu(q)$ of a framenet $(G(q), q)$ as

$$\mu(q) = \frac{1}{|\mathcal{E}_{G(q)}|h_0}[V(q) + \lambda\|\nabla V(q)\|^2] \geq 0 \quad (19)$$

where $\lambda > 0$ is a constant, $V(q)$ is the potential of the α-net, and $h_0 = \psi_\alpha(r - d_\alpha)$. Apparently, the defect factor of any realization of a s_α-net is zero.

5 Obstacle Avoidance using β and γ Agents

In this section, we summarize our approach to flocking with multiple obstacle avoidance for a dynamic α-net that is presented in [3]. Many details including the design of the translational controller for a flock are omitted due to the limitation of space and can be found in [3].

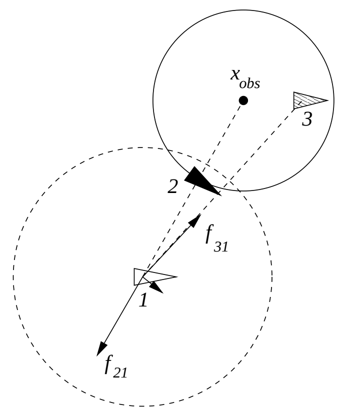


Figure 6: The interaction between an α-agent (agent 1) and the effect of an obstacle represented by a β-agent (agent 2) and a γ-agent (agent 3).

For a net of α-agents we consider the task of moving with a desired group velocity $p_d \neq 0$ along the desired direction $\mathbf{n}_d = p_d/\|p_d\|$ that is a unit vector while avoiding collision to finite number of fixed obstacles. The main assumption on the obstacles is that they are convex and compact sets and their boundaries are closed differentiable Jordan curves in $\mathbb{R}^2$. For simplicity, we assume all obstacles are closed balls and denote them by O_k for $k = 1, \ldots, m$. Let $d(q_i, O_k)$ be the distance between q_i and O_k and $\hat{q}_i^k$ denote the projection of q_i on the boundary of O_k. We refer to an agent with position $\hat{q}_i^k$ as a β*-agent* provided that $d(q_i, O_k) \leq r_0$. For flat obstacles, this projection technique is previously used in [1]. In Figure 6, agent 2 is an example of a β-agent and agent 3 is called a γ-agent that is defined in [3]. Both β and γ agents associated with an α-agent v_i adjacent to an obstacle O_k "disappear" as soon as no points on the boundary of O_k belongs to the neighborhood of node v_i.

The task of β-agents is to repel α-agents. The complete protocol for flocking in presence of obstacles is discussed in [3] and is reduced to interactions among α, β, and γ agents. Figure 7 shows snapshots of the *split/rejoin maneuver* and the *squeezing maneuver* for a group of $n = 100$ agents. These tasks are performed as by-products of flocking in presence of $m = 6$ obstacles. Most agents asymptotically construct a large flock of 95 agents.

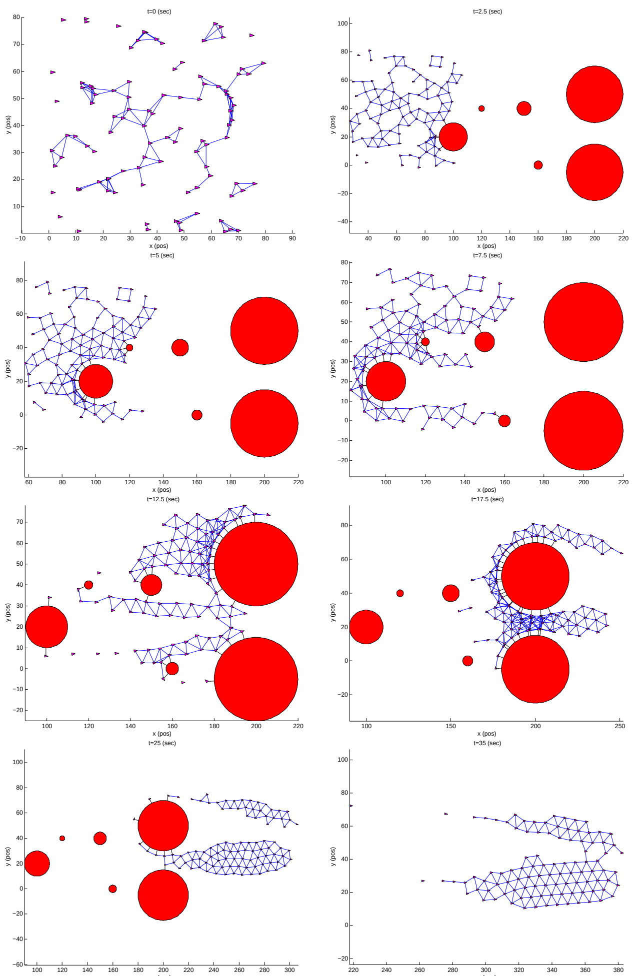


Figure 7: Consecutive snapshots of flocking for a dynamic α-net with $n = 100$ agents in presence of $m = 6$ obstacles and performing split, rejoin, and squeezing maneuvers.

6 Conclusion

In this work, we provided a dynamic graph theoretic framework that enables modeling the flocking of agents in presence of multiple obstacles. We presented formal definitions of nets and flocks as spatially induced graphs. The notion of a framenet was introduced and construction of energy of dynamic α-nets was described. The task of flocking as a group was posed as an individual task of an α-agent and achieved via dissipation of structural energy of a net. We proved that two flocking rules of Reynolds are special cases of a single flocking protocol called the (α, α) protocol. Obstacle avoidance was achieved (informally) by introducing β and γ agents. Simulation results for two challenging tasks of split/rejoin maneuver and squeezing maneuver were presented.

Acknowledgments

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