Reduced magnetohydrodynamic equations with coupled Alfvén and sound wave dynamics

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A set of reduced magnetohydrodynamic equations is described that is appropriate for the simulation of auroral Alfvén waves using curvilinear coordinates. These equations include the parallel electric field, ponderomotive force, diamagnetic drift effects, and gravity and rotation, but do not include the fast mode dynamics. The equations are formulated for multiple species, but quasineutrality is explicitly maintained. The equations have an exact conserved energy. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2786060]

I. INTRODUCTION

Observations from polar orbiting spacecraft suggest that auroral phenomena such as parallel electric fields, accelerated electrons, and uplifted ionospheric ions are caused by Alfvén waves with dispersion resulting from the finite temperature and/or electron inertia of the plasma.[1–3] Numerical simulations of these phenomena have been based mainly on two-fluid equations, including in addition to dispersion the perpendicular displacement current[4–6] and the diamagnetic current and the Alfvén ponderomotive force.[7–10] The displacement current supplants the Alfvén wave polarization current in regions of low plasma density, whereas diamagnetic currents are induced by flute perturbations in the direction of the wave electric drift. The ponderomotive force nonlinearly couples Alfvén waves and ion sound waves when the wave amplitude and wave frequency become relatively large. All of these effects appear to operate in the topside auroral ionosphere and low-altitude magnetosphere.

Our purpose here is to derive a set of equations appropriate for simulating auroral Alfvén waves that is more complete than those used previously, and for which there is an exact conserved energy. When we say that the energy is conserved, we mean that the energy relation of the system may be expressed in a conservative form,

$$\frac{\partial}{\partial t}\mathcal{E} = -\nabla \cdot S. \qquad (1)$$

The energy density ϵ and energy flux $\mathbf{S}$ are given in Sec. III below. A conservative form for the energy equation is especially useful for evaluating power flow through the system and for describing the partitioning and conversion of energy between kinetic, thermal, electromagnetic, and gravitational forms.

As in previous analysis, the equations to be derived here include parallel electric fields due to the finite ion Larmor radius, electron inertia, and electrical resistivity, diamagnetic drift effects, the perpendicular displacement current, and the Alfvén wave ponderomotive force,[8] which is proportional to $\mathbf{B}_\perp \times \partial \mathbf{E}_\perp / \partial t$ in the reduced fluid model. The ponderomotive force is important in driving parallel flows in the auroral region.[11] Multiple species (e.g., H^+, He^+, and O^+), curvilinear geometry, gravitation, and rotation are also allowed. The equations are global, and derivatives on the background magnetic field are retained.

In addition to the usual magnetohydrodynamics (MHD) orderings, $\partial/\partial t \ll \omega_{ci}$ and $\rho_i/L_\perp \ll 1$ in terms of the ion gyrofrequency ω_{ci} and ion gyroradius ρ_i, we further assume that $\partial/\partial t \sim (\rho_i^2/L_\perp^2)\omega_{ci}$ wherein the polarization and diamagnetic drifts are comparable in magnitude and weak in comparison with the dominant $\mathbf{E}\times\mathbf{B}$ drift. The analysis is also restricted to a class of disturbances in which the magnetic compressibility is weak, and the parallel component of the magnetic perturbation may be neglected. For such disturbances, Faraday's law implies that the perpendicular velocity perturbation satisfies $\nabla \cdot \mathbf{v}_\perp \ll \partial/\partial t$ while the solenoidal condition on $\mathbf{B}$ implies that the perpendicular and parallel length scales satisfy $L_\perp \ll L_\parallel$. Magnetic incompressibility does not restrict the gradient $\nabla_\parallel v_\parallel$, so order unity density perturbations are allowed.

These approximations are suitable for describing ultralow frequency electromagnetic power flows from high-altitude magnetospheric dynamos to low-altitude auroral regions when the transverse length scale of the electromagnetic perturbations is sufficiently small. Singly ionized oxygen is the dominant plasma constituent of the topside ionosphere, and its low-altitude, magnetic field-aligned dynamics are strongly constrained by gravity and ambipolar electric fields; thus we have included these effects in the model equations. The topside ionospheric plasma is also weakly collisional, so we also include ion-electron collisions in the description of the parallel motion and the parallel thermal conductivity as given by Braginski.[12] However, the model does not explicitly include ion-neutral collisions, which are important in the E region and bottomside F region of the ionosphere,[13] i.e., the low-altitude end of the physical domain of interest

Although the derived equations are not solved in this paper, low-altitude boundary conditions for auroral applications typically include the effects of the conducting ionosphere. Simple treatments of the ionosphere include a perfectly conducting spherical surface to represent the altitudinally thin E region of the ionosphere, or a finite, uniform constant conducting surface dominated by ion-neutral collisions and characterized by an anisotropic height-integrated conductivity,[13] combined with current continuity. Either type of boundary conditions promotes standing or interfering wave structures; the finite conductivity additionally introduces energy dissipation in the Alfvén wave dynamics. Streltsov and Lotko[7] relax the constant conductivity assumption by including a time-dependent equation for continuity of the height-integrated ion density and, therefore, the height-integrated conductivity, including sources and losses of ionization. This electrodynamically "active" boundary condition leads to the interesting phenomenon of spontaneously generated Alfvén waves via a negative-energy "feedback instability."[14] Because Alfvén and ion sound waves are magnetically guided, the lateral boundary conditions (transverse to the guide field) do not affect the dynamics of disturbances initially localized away from the lateral boundaries. Nevertheless, the lateral boundary conditions must be consistent with the model equations if artificial dynamics are to be avoided.[15]

Due to the magnetic and plasma inhomogeneity, the length scales $L_\perp$ and $L_\parallel$ of initial disturbances vary continuously along the magnetic field. Magnetic guidance of shear Alfvén and ion sound disturbances implies that the perpendicular scale length $L_\perp$ scales as the distance between neighboring field lines, which varies approximately as $B_0^{1/2}$ in terms of the geomagnetic (guide) field. For applications to Alfvén waves, $L_\parallel$ scales locally as the effective parallel wavelength V_A/f, where V_A is the local Alfvén speed and f is the wave frequency. On auroral field lines, V_A varies from a low value of about 1000 km/s in the outer magnetosphere[16] and near the F-region peak in the ionosphere, to a high value that can exceed the speed of light in the very tenuous low-altitude magnetosphere.[17] Relevant frequencies in the MHD range span $f \lesssim 0.1$ Hz. Thus for $f \approx 0.1$ Hz, $L_\perp/L_\parallel$ will remain small along the entire flux tube when the transverse length scale of the disturbance is $\lesssim 1000$ km in the outer magnetosphere. This transverse length scale maps along the flux tube to a value of about 30 km in the ionosphere.

The derivation of the equations given in the next sections follows the analysis of Zeiler *et al.* (Sec. II of Ref. 18), which, in turn, was based on the equations of Braginski.[12] (For other reduced Braginski equations, see the references listed in Ref. 18.) We have written Zeiler *et al.*'s equations in such a way as to allow for a curved background magnetic field. Also, the equations of Zeiler *et al.* are not practically solvable as written since the polarization drift velocity includes a time derivative of the polarization drift velocity, leading to a recursive term. In our formulation, only the lowest-order velocity appears in the polarization drift, and the polarization drift appears only in the current continuity (vorticity) equation (25). (The neglect of the polarization drift in the continuity equation (16) means that the density, and hence the nonlinear Alfvén speed, will not be correctly described for Alfvén waves, but for Alfvén waves the density perturbation will be small. The continuity equation does include the parallel velocity, so large density perturbations can possibly arise from the parallel dynamics.) In addition to terms from Zeiler *et al.*, we have included electron inertia, as well as gravity, rotational forces, and the perpendicular displacement current. (Electron inertia is included in the equations of Zeiler,[19] though not in a manner that exactly conserves energy.) These equations are similar to those recently published by Brizard.[20] His equations are derived from a drift-fluid Lagrangian, but our equations are more easily interpreted and include additional effects such as rotation, gravity, resistivity, and parallel thermal conduction.

II. EQUATIONS

A. Coordinate system and electromagnetic fields

Depending of the application, the equations derived below can be simplified by setting the resistivity η, the parallel thermal conduction $\kappa_\parallel^e$, the gravitation constant G, or the rotational angular frequency Ω_r equal to zero. Further simplifications can be obtained by dropping terms enclosed in the subscripted square brackets that appear in the various reduced equations. For example, if all terms enclosed by $[\cdots]_j$ (with the same subscript j) are dropped, the conservative form of the energy equation derived in Sec. III will be preserved. The $j=1$ terms include finite Larmor radius modifications to the polarization drift;[19] the $j=2$ term comes from the ion heat flux [Eq. (2.3i) with Eq. (2.14) from Ref. 12]; and the $j=3$ terms come from the so-called thermal force [Eqs. (2.2e) and (2.3e) with Eq. (2.9) from Ref. 12].

We assume that the background magnetic field $\boldsymbol{B_0}$ lies in a meridional plane (necessary to define orthogonal coordinates), and that $\boldsymbol{\nabla}\times\boldsymbol{B_0}=0$. That is, there is no current associated with the equilibrium field. [This restriction can be relaxed, but in order to define orthogonal coordinates it is necessary that $\boldsymbol{b_0}\cdot\boldsymbol{J_0}=0$, where $\boldsymbol{b_0}=\boldsymbol{B_0}/B_0$ and $\boldsymbol{J_0}=(c/4\pi)\boldsymbol{\nabla}\times\boldsymbol{B_0}$.[21]] A dipole magnetic field is an example of a field satisfying these conditions. Let s' be the parallel coordinate equal to the distance along a reference field line (it will not in general be exactly equal to the distance s along adjacent field lines). Let

$$\boldsymbol{B}=\boldsymbol{B_0}+\boldsymbol{\nabla}A\times\boldsymbol{\nabla}s', \tag{2}$$

where $\boldsymbol{B}_\perp=\boldsymbol{\nabla}A\times\boldsymbol{\nabla}s'$ (the perpendicular symbol $\perp$ is relative to $\boldsymbol{b_0}$). Let

$$\boldsymbol{b_0}=\frac{\boldsymbol{B_0}}{B_0}, \tag{3a}$$

$$\nabla_{\parallel_0}=\boldsymbol{b_0}\cdot\boldsymbol{\nabla}, \tag{3b}$$

$$\boldsymbol{b}=\boldsymbol{b_0}+\frac{\boldsymbol{B}_\perp}{B_0}=\boldsymbol{b_0}+\boldsymbol{b}_\perp, \tag{3c}$$

$$\nabla_\parallel=\boldsymbol{b}\cdot\boldsymbol{\nabla}. \tag{3d}$$

Note then that $b=\sqrt{\boldsymbol{b}\cdot\boldsymbol{b}}=\sqrt{1+B_\perp^2/B_0^2}\neq 1$. (But $B_\perp^2/B_0^2$ is assumed to be small.)

Let

$$\boldsymbol{E} = -\nabla\phi - \frac{1}{c}\frac{\partial A}{\partial t}\nabla s' = -\nabla\phi - \boldsymbol{b}_0\frac{1}{c}\frac{\partial A}{\partial t}\nabla_{\|_0} s'. \tag{4}$$

Note then that $\nabla\cdot\boldsymbol{B}=0$ and Faraday's law holds exactly. (Note that $\nabla s'=\boldsymbol{b_0}$ on the reference field line, where $\nabla_{\|_0}s'=1$, and that this will be approximately true on adjacent field lines.)

The current is

$$\boldsymbol{J} = \frac{c}{4\pi}\nabla\times\boldsymbol{B}, \tag{5}$$

and the parallel ($\boldsymbol{b_0}$ component) from Eq. (2) is

$$J_{\|_0} = \boldsymbol{b}_0\cdot\boldsymbol{J} = \frac{c}{4\pi}\boldsymbol{b}_0\cdot\nabla\times(\nabla A\times\nabla s')$$
$$= -\frac{c}{4\pi}[\nabla_\perp\cdot(\nabla_\perp A\nabla_{\|_0}s') + \nabla_{\|_0}s'\boldsymbol{\kappa}\cdot\nabla_\perp A], \tag{6}$$

where $\boldsymbol{\kappa}\equiv\boldsymbol{b_0}\cdot\nabla\boldsymbol{b_0}$ is the curvature vector.

B. Velocities and convective derivatives

We define several forms for the ion velocity that are used in different equations,

$$\boldsymbol{v}_{i_0} = v_{\|i}\boldsymbol{b} + \boldsymbol{v_E}, \tag{7a}$$

$$\boldsymbol{v}_{i_1} = v_{\|i}\boldsymbol{b} + \boldsymbol{v_E} + \boldsymbol{v}_{d_i}, \tag{7b}$$

$$\boldsymbol{v}_{i_2} = v_{\|i}\boldsymbol{b} + \boldsymbol{v_E} + \boldsymbol{v}_{d_i} + \boldsymbol{v}_{G_i} + \boldsymbol{v}_{\mathrm{pol}_i}, \tag{7c}$$

in order of increasing accuracy, where

$$\boldsymbol{v_E} = \frac{c\boldsymbol{E}\times\boldsymbol{b_0}}{B_0}, \tag{8a}$$

$$\boldsymbol{v}_{d_i} = \frac{c\boldsymbol{b_0}\times\nabla p_i}{q_i n_i B_0}, \tag{8b}$$

$$\boldsymbol{v}_{G_i} = \boldsymbol{v}_{g_i} + \boldsymbol{v}_{c_i}, \tag{8c}$$

$$\boldsymbol{F}_{G_i} = \boldsymbol{F}_{g_i} + \boldsymbol{F}_{c_i}, \tag{8d}$$

$$\boldsymbol{v}_{g_i} = \frac{c\boldsymbol{F}_{g_i}\times\boldsymbol{b_0}}{q_i B_0}, \tag{8e}$$

$$\boldsymbol{F}_{g_i} = -m_i\nabla\Phi_g, \tag{8f}$$

$$\Phi_g = -\frac{M_E G}{R} - \frac{1}{2}\Omega_r^2 R^2\sin^2\theta, \tag{8g}$$

$$\boldsymbol{v}_{c_i} = \frac{c\boldsymbol{F}_{c_i}\times\boldsymbol{b_0}}{q_i B_0}, \tag{8h}$$

$$\boldsymbol{F}_{c_i} = -2m_i\boldsymbol{\Omega_r}\times\boldsymbol{v}_{i_1}, \tag{8i}$$

and the ion polarization drift is

$$\boldsymbol{v}_{\mathrm{pol}_i} = \frac{\boldsymbol{b_0}}{\omega_{ci}}\times\frac{d^i}{dt}\boldsymbol{v}_{i_0} + \frac{1}{n_i m_i\omega_{ci}}\boldsymbol{b_0}\times\left[p_i\left(\nabla\times\left(\frac{\boldsymbol{b_0}}{\omega_{ci}}\right)\right)\cdot\nabla\boldsymbol{v}_{i_0}\right] + \left[\frac{1}{n_i m_i\omega_{ci}}\left(\boldsymbol{b_0}\times\nabla_\perp\left(\frac{p_i}{2\omega_{ci}}\nabla\cdot(\boldsymbol{b_0}\times\boldsymbol{v_E})\right) - \nabla_\perp\left(\frac{p_i}{2\omega_{ci}}\nabla_\perp\cdot\boldsymbol{v_E}\right)\right)\right]_1, \tag{9}$$

where

$$\frac{d^i}{dt} = \frac{\partial}{\partial t} + \boldsymbol{v}_{i_0}\cdot\nabla = \frac{\partial}{\partial t} + (v_{\|i}\boldsymbol{b} + \boldsymbol{v_E})\cdot\nabla, \tag{10}$$

and where $\omega_{ci}=q_iB_0/(m_ic)$. (Again, terms in square brackets are not necessary for energy conservation.) Our plan is to have multiple ion species, so the "i" subscript serves as a species index. In Eq. (8), $\boldsymbol{v_E}$ is the $\boldsymbol{E}\times\boldsymbol{B}$ velocity, $\boldsymbol{v}_{d_i}$ is the ion diamagnetic drift, $\boldsymbol{F}_{g_i}$ and $\boldsymbol{v}_{g_i}$ are the gravitational force and gravitational drift velocity, respectively, including the pseudo-gravitational effect of rotation, M_E is the mass of the Earth, G is the universal gravitation constant, R is the geocentric distance, $\boldsymbol{\Omega_r}$ is the rotation frequency vector (with the direction of the axis of rotation), θ is the polar angle from the rotation axis, and $\boldsymbol{F}_{c_i}$ and $\boldsymbol{v}_{c_i}$ are the Coriolis force and Coriolis drift velocity, respectively. For a detailed explanation of the ion gyroviscous terms in Eq. (9) and other equations, see the references listed in Ref. 18.

The electron velocity is

$$\boldsymbol{v}_{e_0} = v_{\|e}\boldsymbol{b} + \boldsymbol{v_E}, \tag{11a}$$

$$\boldsymbol{v}_{e_1} = v_{\|e}\boldsymbol{b} + \boldsymbol{v_E} + \boldsymbol{v}_{d_e}, \tag{11b}$$

$$\boldsymbol{v}_{e_q} = \left[\sum_i (q_i n_i\boldsymbol{v}_{i_1}) - \boldsymbol{J}\right]\Big/(en_e), \tag{11c}$$

where

$$v_{\|e} = \left[\sum_i (q_i n_i v_{\|i}) - J_{\|0}\right]\Big/(en_e), \tag{12}$$

and the electron diamagnetic drift velocity is

$$\boldsymbol{v}_{d_e} = -c\boldsymbol{b_0}\times\nabla p_e/(en_eB_0). \tag{13}$$

The electron density is

$$n_e = \sum_i q_i n_i/e \tag{14}$$

(assuming quasineutrality), and we define

$$\frac{d^e}{dt} = \frac{\partial}{\partial t} + \boldsymbol{v}_{e_q}\cdot\nabla. \tag{15}$$

Note that the perpendicular current appears only in $\boldsymbol{v}_{e_q}$. (The use of $\mathbf{J}_\perp$ in $\boldsymbol{v}_{e_q}$ is necessary for d^e/dt [Eq. (15)], which appears in Eq. (24). Then the convective derivative in Eq. (24) is consistent with our definition of n_e in Eq. (14), allowing us to use Eq. (39) to derive energy conservation.)

C. Continuity and pressure equations

The ion continuity equation (for each species) is

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \boldsymbol{v}_{i_1}) = 0. \tag{16}$$

The ion pressure equation is

$$\frac{3}{2}\frac{d^i p_i}{dt} + \frac{5}{2} p_i \nabla \cdot (\boldsymbol{v}_{i_0}) + p_i \nabla \cdot (\boldsymbol{v}_{G_i}) + \left[\frac{5}{2}\frac{c}{e}\left(\nabla \times \frac{\boldsymbol{b}_0}{B_0}\right) \cdot \nabla(p_i T_i)\right]_2 = 0, \tag{17}$$

where $T_i = p_i/n_i$.

The electron pressure equation is

$$\frac{3}{2}\frac{d^e p_e}{dt} + \frac{3}{2} p_e \nabla \cdot (\boldsymbol{v}_{e_q}) + p_e \nabla \cdot (\boldsymbol{v}_{e_0}) - \eta_\parallel J_{\parallel 0}^2 + [-0.71 T_e \nabla \cdot (\boldsymbol{b} n_e v_{\parallel e})]_3 = \nabla \cdot (\boldsymbol{b} \kappa_\parallel^e \nabla_\parallel T_e), \tag{18}$$

where $\eta_\parallel$ is the parallel resistivity, $T_e = p_e/n_e$, and $\kappa_\parallel^e$ is the electron thermal conductivity. (If the term in square brackets marked with subscript "3" is kept, the corresponding term in the electron momentum equation should also be kept.) We have assumed that resistive heating dominantly heats the electrons, which is approximately true because of their lower mass.

D. Momentum equations

The ion parallel momentum equation is derived from Ref. 18,

$$m_i n_i \left(\frac{\partial}{\partial t} + \boldsymbol{v}_i \cdot \nabla\right) \boldsymbol{v}_i = -\nabla p_i - (\nabla \cdot P_i) + q_i n_i \times \left(\boldsymbol{E} - \eta_\parallel J_{\parallel 0} \boldsymbol{b}_0 + \frac{\boldsymbol{v}_i \times \boldsymbol{B}}{c}\right) + n_i \boldsymbol{F}_{G_i}, \tag{19}$$

where $\boldsymbol{v}_i$ indicates the full ion velocity, and the divergence of the pressure tensor including finite Larmor radius effects is

$$(\nabla \cdot P_i) = -m_i n_i \boldsymbol{v}_{d_i} \cdot \nabla \boldsymbol{v}_i + p_i \left(\nabla \times \frac{\boldsymbol{b}}{\omega_{ci}}\right) \cdot \nabla \boldsymbol{v}_i + \left[\nabla_\perp \left(\frac{p_i}{2\omega_{ci}} \nabla \cdot (\boldsymbol{b} \times \boldsymbol{v}_i)\right) + \boldsymbol{b} \times \nabla \left(\frac{p_i}{2\omega_{ci}} \nabla_\perp \cdot \boldsymbol{v}_i\right)\right]_1, \tag{20}$$

where $\nabla_\perp$ here (only) is perpendicular to $\boldsymbol{b}$. [Note that Eq. (20) was used already for $\boldsymbol{v}_{\mathrm{pol}_i}$, approximating $\boldsymbol{b}$ by $\boldsymbol{b}_0$ and $\boldsymbol{v}_i$ by $\boldsymbol{v}_{i_0}$ in the last two terms on the right side of Eq. (20).] In order to make further progress, we approximate the first $\boldsymbol{v}_i$ in the convective term on the left side of Eq. (19) by $\boldsymbol{v}_{i_1}$, and the second $\boldsymbol{v}_i$ in the same term by $\boldsymbol{v}_{i_0}$. [There is then a cancellation involving the first term on the right side of Eqs. (20).[19]] We also use $\boldsymbol{v}_{i_0}$ for $\boldsymbol{v}_i$ in the first two terms on the right side of Eq. (20). (The time derivative acting on the velocity yields the polarization drift, so we are excluding here the part of the polarization drift with the time derivative acting on the diamagnetic drift velocity. This could be added with some increase in complexity.) Dotting Eq. (19) with $\boldsymbol{b}$, we get

$$m_i n_i \boldsymbol{b} \cdot \frac{d^i \boldsymbol{v}_{i_0}}{dt} = -\nabla_\parallel p_i - p_i \boldsymbol{b} \cdot \left[\left(\nabla \times \frac{\boldsymbol{b}_0}{\omega_{ci}}\right) \cdot \nabla \boldsymbol{v}_{i_0}\right] + q_i n_i (E_\parallel - \eta_\parallel J_{\parallel 0}) + n_i \boldsymbol{b} \cdot \boldsymbol{F}_{G_i}. \tag{21}$$

We can expand

$$\boldsymbol{b} \cdot \frac{d^i \boldsymbol{v}_{i_0}}{dt} = b^2 \frac{d^i v_{\parallel i}}{dt} + v_{\parallel i}\left(\boldsymbol{b} \cdot \frac{d^i \boldsymbol{b}_0}{dt} + \boldsymbol{b} \cdot \frac{\partial \boldsymbol{b}_\perp}{\partial t} + \boldsymbol{b} \cdot (\boldsymbol{v}_{i_0} \cdot \nabla \boldsymbol{b}_\perp)\right) + \boldsymbol{b} \cdot \frac{d^i \boldsymbol{v}_E}{dt}. \tag{22}$$

The $\boldsymbol{b} \cdot d^i \boldsymbol{b}_0/dt$ term is equal to $\boldsymbol{b}_\perp \cdot (\boldsymbol{\kappa} v_{\parallel i} + \boldsymbol{v}_E \cdot \nabla \boldsymbol{b}_0)$, where $\boldsymbol{\kappa} = \boldsymbol{b}_0 \cdot \nabla \boldsymbol{b}_0$ is the curvature of the background magnetic field. The $\boldsymbol{b} \cdot \partial \boldsymbol{b}_\perp/\partial t$ term can be rewritten as $-(c/B_0)\boldsymbol{b}_\perp \cdot \nabla \times \boldsymbol{E}$ using Faraday's law, $\partial \boldsymbol{B}/\partial t = \partial \boldsymbol{B}_\perp/\partial t = -c\nabla \times \boldsymbol{E}$. Then we can rewrite Eq. (21) as

$$m_i n_i b^2 \frac{d^i v_{\parallel i}}{dt} = -m_i n_i v_{\parallel i}\left(\boldsymbol{b}_\perp \cdot (\boldsymbol{\kappa} v_{\parallel i} + \boldsymbol{v}_E \cdot \nabla \boldsymbol{b}_0) - \frac{c}{B_0}\boldsymbol{b}_\perp \cdot \nabla \times \boldsymbol{E} + \boldsymbol{b} \cdot (\boldsymbol{v}_{i_0} \cdot \nabla \boldsymbol{b}_\perp)\right) - m_i n_i \boldsymbol{b} \cdot \frac{d^i \boldsymbol{v}_E}{dt} - \nabla_\parallel p_i - p_i \boldsymbol{b} \cdot \left[\left(\nabla \times \frac{\boldsymbol{b}_0}{\omega_{ci}}\right) \cdot \nabla \boldsymbol{v}_{i_0}\right] + q_i n_i (E_\parallel - \eta_\parallel J_{\parallel 0}) + n_i \boldsymbol{b} \cdot \boldsymbol{F}_{G_i}. \tag{23}$$

The $m_i n_i \boldsymbol{b} \cdot d^i \boldsymbol{v}_E/dt$ term includes the so-called ponderomotive force $\propto (\partial \boldsymbol{E}_\perp/\partial t) \times \boldsymbol{B}_\perp$.

The electron parallel momentum equation is simpler, since we can neglect most of the finite Larmor radius (FLR) terms as well as gravity and the ponderomotive terms.

$$m_e n_e \frac{d^e v_{\parallel e}}{dt} = -\nabla_\parallel p_e - e n_e (E_\parallel - \eta_\parallel J_{\parallel 0}) - [0.71 n_e \nabla_\parallel T_e]_3, \tag{24}$$

where $\omega_{ce} = eB_0/(m_e c)$, and the term in brackets must be kept if the corresponding term is kept in the electron pressure equation.

E. Current continuity equation

Finally, the current continuity (vorticity) equation is

$$\nabla \cdot (q_i n_i \boldsymbol{v}_{i_2} - e n_e \boldsymbol{v}_{e_1}) + \nabla \cdot \left(\frac{1}{4\pi}\frac{\partial \boldsymbol{E}_\perp}{\partial t}\right) = 0, \tag{25}$$

where we have included the displacement current in the perpendicular electric field. The parallel displacement current divided by the perpendicular displacement current $\sim (E_\parallel/L_\parallel)/(E_\perp/L_\perp) \sim (E_\parallel/E_\perp)(L_\perp/L_\parallel)$. Both $E_\parallel/E_\perp$ and $L_\perp/L_\parallel$ are small (as experimentally observed), so the parallel displacement current should be negligible compared to the perpendicular displacement current. [Futhermore, keeping the parallel displacement current $\propto \nabla \cdot (\boldsymbol{b_0}\partial E_\parallel/\partial t)$ would lead to undesirable (high-frequency) plasma wave oscillations along the magnetic field.]

F. Summary of key equations

Ion continuity equation (16)

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \boldsymbol{v}_{i_1}) = 0, \tag{26}$$

with the definition in Eq. (7b).

Electron density from the quasineutrality equation (14)

$$n_e = \sum_i q_i n_i / e. \tag{27}$$

Ion pressure equation (17)

$$\frac{3}{2}\frac{d^i p_i}{dt} + \frac{5}{2}p_i \nabla \cdot (\boldsymbol{v}_{i_0}) + p_i \nabla \cdot (\boldsymbol{v}_{G_i}) + \left[\frac{5}{2}\frac{c}{e}\left(\nabla \times \frac{\boldsymbol{b_0}}{B_0}\right)\cdot \nabla(p_i T_i)\right]_2 = 0, \tag{28}$$

with definitions in Eqs. (10), (7a), and (8c).

Electron pressure equation (18)

$$\frac{3}{2}\frac{d^e p_e}{dt} + \frac{3}{2}p_e \nabla \cdot (\boldsymbol{v}_{e_q}) + p_e \nabla \cdot (\boldsymbol{v}_{e_0}) - \eta_\parallel J_{\parallel 0}^2 + [-0.71 T_e \nabla \cdot (\boldsymbol{b} n_e v_{\parallel e})]_3 = \nabla \cdot (\boldsymbol{b}\kappa_\parallel^e \nabla_\parallel T_e), \tag{29}$$

with definitions in Eqs. (15), (11c), (11a), and (6).

Ion momentum equation (23)

$$\begin{aligned} m_i n_i b^2 \frac{d^i v_{\parallel i}}{dt} = {} & - m_i n_i v_{\parallel i}\left(\boldsymbol{b}_\perp \cdot (\boldsymbol{\kappa} v_{\parallel i} + \boldsymbol{v_E} \cdot \nabla \boldsymbol{b_0}) - \frac{c}{B_0}\boldsymbol{b}_\perp \cdot \nabla \times \boldsymbol{E} + \boldsymbol{b}\cdot(\boldsymbol{v}_{i_0}\cdot \nabla \boldsymbol{b}_\perp)\right) \\ & - m_i n_i \boldsymbol{b}\cdot \frac{d^i \boldsymbol{v_E}}{dt} - \nabla_\parallel p_i \\ & - p_i \boldsymbol{b}\cdot\left[\left(\nabla \times \frac{\boldsymbol{b_0}}{\omega_{ci}}\right)\cdot \nabla \boldsymbol{v}_{i_0}\right] + q_i n_i (E_\parallel - \eta_\parallel J_{\parallel 0}) \\ & + n_i \boldsymbol{b}\cdot \boldsymbol{F}_{G_i}, \end{aligned} \tag{30}$$

with definitions in Eqs. (10), (8a), (7a), and (6).

Electron momentum equation

$$m_e n_e \frac{d^e v_{\parallel e}}{dt} = - \nabla_\parallel p_e - e n_e (E_\parallel - \eta_\parallel J_{\parallel 0}) - [0.71 n_e \nabla_\parallel T_e]_3. \tag{31}$$

Current continuity (vorticity) equation (25)

$$\nabla \cdot (q_i n_i \boldsymbol{v}_{i_2} - e n_e \boldsymbol{v}_{e_1}) + \nabla \cdot \left(\frac{1}{4\pi}\frac{\partial \boldsymbol{E}_\perp}{\partial t}\right) = 0, \tag{32}$$

with definitions in Eqs. (7c) and (11b).

III. ENERGY CONSERVATION

The derivation of energy conservation is similar to that of Zeiler *et al.*[18] To derive an energy, we first multiply the ion parallel momentum equation (21) by $v_{\parallel i}$, the electron momentum equation (24) by $v_{\parallel e}$, and add them together. For multiple ion species, there is an implied sum over the i index,

$$\begin{aligned} & + m_i n_i v_{\parallel i} \boldsymbol{b}\cdot \frac{d^i \boldsymbol{v}_{i_0}}{dt} + p_i v_{\parallel i}\boldsymbol{b}\cdot\left[\left(\nabla \times \frac{\boldsymbol{b_0}}{\omega_{ci}}\right)\cdot \nabla \boldsymbol{v}_{i_0}\right] \\ & + n_e \frac{d^e}{dt}\left(\frac{1}{2}m_e v_{\parallel e}^2\right) + v_{\parallel i}\nabla_\parallel p_i + v_{\parallel e}\nabla_\parallel p_e \\ & + m_i n_i v_{\parallel i}\boldsymbol{b}\cdot \nabla \Phi_g + 2 m_i n_i v_{\parallel i}\boldsymbol{b}\cdot(\Omega_r \times \boldsymbol{v}_{i_1}) \\ & - J_{\parallel 0}(E_\parallel - \eta_\parallel J_{\parallel 0}) + [0.71 n_e v_{\parallel e}\nabla_\parallel T_e]_3 = 0. \end{aligned} \tag{33}$$

We then add the ion pressure equation (17) and the electron pressure equation (18) together,

$$\begin{aligned} & \frac{\partial}{\partial t}\left(\frac{3}{2}p_i + \frac{3}{2}p_e\right) - \boldsymbol{v}_{i_0}\cdot \nabla p_i - \boldsymbol{v}_{e_0}\cdot \nabla p_e + p_i \nabla \cdot \boldsymbol{v}_{G_i} - \eta_\parallel J_{\parallel 0}^2 \\ & + \left[\frac{5}{2}\frac{c}{e}\left(\nabla \times \frac{\boldsymbol{b_0}}{B_0}\right)\cdot \nabla(p_i T_i)\right]_2 \\ & + [0.71 T_e \nabla \cdot (\boldsymbol{b} n_e v_{\parallel e})]_3 \\ & = - \nabla \cdot \left\{\frac{5}{2}p_i \boldsymbol{v}_{i_0} + p_e\left(\frac{3}{2}\boldsymbol{v}_{e_q} + \boldsymbol{v}_{e_0}\right) - \boldsymbol{b}\kappa_\parallel^e \nabla_\parallel T_e\right\}. \end{aligned} \tag{34}$$

Using Faraday's law,

$$\frac{\partial}{\partial t}\left(\frac{\boldsymbol{B}_\perp^2}{8\pi}\right) - \frac{1}{c}\nabla_{\parallel 0} s' J_{\parallel 0}\frac{\partial A}{\partial t} = - \nabla \cdot \left\{- \nabla_\perp A \frac{1}{4\pi}(\nabla_{\parallel 0} s')^2 \frac{\partial A}{\partial t}\right\}. \tag{35}$$

Then we multiply the current continuity equation by ϕ,

$$\begin{aligned} & \frac{\partial}{\partial t}\left(\frac{E_\perp^2}{8\pi}\right) - J_{\parallel 0}\boldsymbol{b}\cdot \nabla \phi + \boldsymbol{v_E}\cdot\left\{m_i n_i \frac{d^i \boldsymbol{v}_{i_0}}{dt} + m_i n_i (\nabla \Phi_g + 2\Omega_r \times \boldsymbol{v}_{i_1}) + p_i\left[\nabla \times \left(\frac{\boldsymbol{b_0}}{\omega_{ci}}\right)\right]\cdot \nabla \boldsymbol{v}_{i_0} + \nabla(p_i + p_e)\right\} \\ & = - \nabla \cdot \left\{\phi\left(J_{\parallel 0}\boldsymbol{b} + q_i n_i (\boldsymbol{v}_{d_i} + \boldsymbol{v}_{\mathbf{pol}_i} + \boldsymbol{v}_{G_i}) - e n_e \boldsymbol{v}_{d_e} + \frac{1}{4\pi}\frac{\partial \boldsymbol{E}_\perp}{\partial t}\right) \times \left[\frac{p_i}{2\omega_{ci}}[\boldsymbol{v_E}\nabla \cdot (\boldsymbol{b_0}\times \boldsymbol{v_E}) - (\boldsymbol{b_0}\times \boldsymbol{v_E})\nabla_\perp \cdot \boldsymbol{v_E}]\right]_1\right\}. \end{aligned} \tag{36}$$

Now we make use of several useful identities (the volume integrated forms of the ion equations are given in Ref. 18),

$$p_i\left[\nabla \times \left(\frac{\boldsymbol{b_0}}{\omega_{ci}}\right)\right]\cdot \nabla f = m_i n_i \boldsymbol{v}_{d_i}\cdot \nabla f - \nabla \cdot \left(p_i \nabla f \times \frac{\boldsymbol{b_0}}{\omega_{ci}}\right), \tag{37}$$

$$n_i\left(\frac{d^i}{dt}+\boldsymbol{v}_{d_i}\cdot\boldsymbol{\nabla}\right)f=\frac{\partial(n_if)}{\partial t}+\boldsymbol{\nabla}\cdot(\boldsymbol{v}_{i_1}n_if), \tag{38}$$

using the continuity equation (16), and

$$n_e\frac{d^e}{dt}f=\frac{\partial(n_ef)}{\partial t}+\boldsymbol{\nabla}\cdot(\boldsymbol{v}_{e_q}n_ef), \tag{39}$$

using the effective electron continuity equation implied by quasineutrality [Eq. (14)].

Adding Eqs. (33)–(36), we find Eq. (1) with

$$\mathcal{E}=\frac{m_in_i}{2}(v_{\|i}\boldsymbol{b}+\boldsymbol{v}_E)^2+\frac{m_en_e}{2}v_{\|e}^2+\frac{3}{2}(p_i+p_e)+\frac{1}{8\pi}(\boldsymbol{B}_\perp^2+\boldsymbol{E}_\perp^2)+m_in_i\phi_G, \tag{40}$$

and the energy flux $\boldsymbol{S}$ is

$$\boldsymbol{S}=\frac{1}{2}m_in_i|\boldsymbol{v}_{i_0}|^2\boldsymbol{v}_{i_1}+\frac{1}{2}m_en_e|v_{\|e}|^2\boldsymbol{v}_{e_q}+\left[\frac{p_i}{2\omega_{ci}}[\boldsymbol{v}_E\boldsymbol{\nabla}\cdot(\boldsymbol{b}_0\times\boldsymbol{v}_E)-(\boldsymbol{b}_0\times\boldsymbol{v}_E)\boldsymbol{\nabla}\cdot\boldsymbol{v}_E]\right]_1+\frac{5}{2}p_i\boldsymbol{v}_{i_0}+p_i\boldsymbol{v}_{G_i}+\frac{p_i}{2}\frac{\boldsymbol{b}_0}{\omega_{ci}}\times\boldsymbol{\nabla}|\boldsymbol{v}_{i_0}|^2+\left[\frac{5}{2}\frac{c}{eB_0}\boldsymbol{b}_0\times\boldsymbol{\nabla}(p_iT_i)\right]_2+p_e\left(\boldsymbol{v}_{e_0}+\frac{3}{2}\boldsymbol{v}_{e_q}\right)-\boldsymbol{b}\kappa_\|^e\nabla_\|T_e+[0.71n_eT_ev_{\|e}\boldsymbol{b}]_3-\frac{1}{4\pi}(\nabla_{\|0}s')^2\frac{\partial A}{\partial t}\boldsymbol{\nabla}_\perp A+\phi\left(J_{\|0}\boldsymbol{b}+q_in_i(\boldsymbol{v}_{d_i}+\boldsymbol{v}_{\mathrm{pol}_i}+\boldsymbol{v}_{G_i})-en_e\boldsymbol{v}_{d_e}+\frac{1}{4\pi}\frac{\partial\boldsymbol{E}_\perp}{\partial t}\right). \tag{41}$$

From Eq. (40), we can see that the energy density is composed of terms representing the kinetic energy, the thermal energy, the electromagnetic field energy, and the gravitational potential energy. If the boundary conditions are periodic or such that $\boldsymbol{S}\cdot\hat{\boldsymbol{n}}=0$, where $\hat{\boldsymbol{n}}$ is the boundary normal, the total energy (integrated over the simulation volume) will be conserved. If the boundary condition is not periodic and $\boldsymbol{S}\cdot\hat{\boldsymbol{n}}\neq 0$, an accurate evaluation of the simulation energy will have to take account of the flux of energy through the boundary.

There are a number of possible variations in the equations that can be considered. A possible simplification is to neglect the convective time derivative of $\boldsymbol{b}_0$ in Eq. (21) (leading to a term proportional to the curvature) by separating the terms acting on $\boldsymbol{b}_0v_{\|i}$ from those acting on $\boldsymbol{b}_\perp v_{\|i}+\boldsymbol{v}_e$. There are also some generalizations leading to more complexity. The $\boldsymbol{b}_0\times$ terms can be generalized to $\boldsymbol{b}\times$ without any additional coupling of equations because we can use Faraday's law to evaluate partial time derivatives acting on $\boldsymbol{B}_\perp$. The polarization drift can be extended to include $d\boldsymbol{v}_{d_i}/dt$. Then the $p_i\boldsymbol{\nabla}\cdot\boldsymbol{v}_i$ term in the ion pressure equation must include the polarization drift, and the energy density will include a term $(1/2)m_in_i(v_{\|i}\boldsymbol{b}+\boldsymbol{v}_e+\boldsymbol{v}_{d_i})^2$. Another straightforward modification would be to make the pressure anisotropic using equations based on those of Chew, Goldberger, and Low[22] (see Ref. 20). The effects of ion cyclotron waves could be included using the approach of Denton and Lyon.[23] However, correct treatment of the parallel dynamics including Landau damping would require a gyrofluid approach.[24]

IV. SOLVING THE EQUATIONS

The ion momentum equation (for each species) has the form

$$a_{vi-vi}\frac{\partial v_{\|i}}{\partial t}+\boldsymbol{a}_{\boldsymbol{vi}-\phi}\cdot\boldsymbol{\nabla}_\perp\frac{\partial\phi}{\partial t}+a_{vi-A}\frac{\partial A}{\partial t}=S_{vi}, \tag{42}$$

where a_{vi-vi}, $\boldsymbol{a}_{\boldsymbol{vi}-\phi}$, and a_{vi-A} are fields, and S_{vi} collects all the explicit terms. The term with $\partial\phi/\partial t$ comes from the ponderomotive force. The term with $\partial A/\partial t$ comes from $E_\|$. The electron momentum equation has the form

$$a_{ve-J}\frac{\partial J_{\|0}}{\partial t}+a_{ve-A}\frac{\partial A}{\partial t}+\sum_i a_{ve-vi_i}\frac{\partial v_{\|i}}{\partial t}=S_{ve}. \tag{43}$$

Here $\partial J_{\|0}(A)/\partial t$ is the largest term from $\partial v_{\|e}/\partial t$. The sum with $\partial v_{\|i}/\partial t$ also comes from $\partial v_{\|e}/\partial t$ because of the definition of $v_{\|e}$ in terms of $J_{\|0}$ [Eq. (24)]. [This relation is used in several steps deriving energy conservation, for instance summing the $E_\|$ terms from the momenta equations to get $J_{\|0}(E_\|-\eta_\|J_{\|0})$ in Eq. (33).] The term with $\partial A/\partial t$ comes from $E_\|$. The vorticity equation is of the form

$$\boldsymbol{\nabla}_\perp\cdot\left(a_{\mathrm{vor}-\phi}\boldsymbol{\nabla}_\perp\frac{\partial\phi}{\partial t}+\sum_i\boldsymbol{a}_{\boldsymbol{vor}-\boldsymbol{vi}_i}\frac{\partial v_{\|i}}{\partial t}\right)=S_{\mathrm{vor}}. \tag{44}$$

Here the $\partial\phi/\partial t$ term comes from the $d\boldsymbol{v}_e/dt$ part of the polarization drift and the displacement current (collected together into one term), and the $\partial v_{\|i}/\partial t$ term comes from the $d(v_\|\boldsymbol{b}_\perp)/dt$ part of the polarization drift. The equations can be solved by substituting $\partial v_{\|i}/\partial t$ from Eq. (42) into Eqs. (43) and (44), yielding two coupled equations for A and ϕ. Having the solutions for A and ϕ, one can substitute back into Eq. (42) to evolve $v_{\|i}$.

V. LINEAR DISPERSION RELATION

For a two-component plasma in a homogeneous magnetic field, the linear dispersion relation of our equations is

$$\left(V-\frac{1}{1+\epsilon_c^2}\right)[V(1+\epsilon_m)-\beta'_{\text{tot}}] + \epsilon_m V^2 K - V(\beta'_e+\epsilon_m\beta'_i)K + \beta'_e\beta'_i K = 0, \tag{45}$$

where

$$V \equiv \left(\frac{\omega/k_\parallel}{V_A}\right)^2, \tag{46a}$$

$$K \equiv \left(k_\perp \frac{c}{\omega_{pi}}\right)^2, \tag{46b}$$

$$\epsilon_c \equiv \frac{V_A}{c}, \tag{46c}$$

$$\epsilon_m \equiv \frac{m_e}{m_i}, \tag{46d}$$

$$\beta'_s \equiv \frac{5\beta_s}{6} = \frac{\gamma\beta_s}{2}, \quad \text{with } \gamma = \frac{5}{3}, \tag{46e}$$

$$\beta'_{\text{tot}} \equiv \beta'_e + \beta'_i, \tag{46f}$$

where $V_A \equiv B_0/\sqrt{4\pi m_i n_i}$ is the Alfvén speed, $\omega_{pi} \equiv \sqrt{4\pi n_i q_i^2/m_i}$ is the ion plasma frequency, and $\beta_s = 8\pi p_s/B_0^2$ is the plasma beta for species s (=i or e for ions or electrons). As can be seen from Eqs. (46a) and (46b), V is the normalized ω^2, and K is the normalized $k_\perp^2$. The first line of Eq. (45) (neglecting the terms proportional to K) yields the Alfvén wave ($V\sim 1$) and the sound wave ($V\sim\beta'_{\text{tot}}$, the normalized sound speed squared) solutions. Terms in the dispersion relation result from the product of two terms. The $\epsilon_m V^2 K$ term in Eq. (45) $[\propto (k_\perp c/\omega_{pe})^2]$ comes from the combination of electron inertia and ion inertia. The $V(\beta'_e + \epsilon_m\beta'_i)K$ term $\sim V\beta'_e K$ if $\beta'_e > \epsilon_m \beta'_i$, and $V\beta'_e K \sim (k_\perp \rho_{se})^2$, where $\rho_{se} = c_{se}/\Omega_{ci}$ and $c_{se} = \sqrt{\gamma T_e/m_i}$ is the ion sound speed based on T_e. This term comes from the combination of the electron parallel pressure gradient and ion inertia. The last term comes from the combination of the electron parallel pressure gradient and the ion parallel pressure gradient. [Equation (45) is nearly identical to the corresponding expression for the equations of Ref. 20. In that case, the ϵ_c^2 term should be replaced by $\epsilon_m + \epsilon_c^2$.]

VI. SUMMARY

Starting from the equations of Zeiler *et al.*,[18] we have derived a set of equations for the simulation of coupled Alfvén and sound waves along magnetospheric field lines. These equations include parallel electric fields, the ponderomotive force, and the effects of field line curvature, rotation, and gravity. We have shown that there is a conserved energy given appropriate boundary conditions, and we have given the energy flux. We discussed how the equations can be solved in a numerical code. We have also given the linear dispersion relation for a homogeneous plasma, which shows that there are Alfven and sound wave solutions coupled by finite kc/ω_{pi}.

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