Values of the Dot Product Between Fundamental Unit Vectors in Cartesian, Cylindrical, and Spherical Coordinate Systems.

	$\hat{\mathbf{x}}$	$\hat{\mathbf{y}}$	$\hat{\mathbf{z}}$	$\hat{\mathbf{r}}$	$\hat{\boldsymbol{\theta}}$	$\hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}$	$\hat{\boldsymbol{\phi}}$	$\hat{\mathbf{z}}$
$\hat{\mathbf{x}}$	1	0	0	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$	$\cos\phi$	$-\sin\phi$	0
$\hat{\mathbf{y}}$	0	1	0	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$	$\sin\phi$	$\cos\phi$	0
$\hat{\mathbf{z}}$	0	0	1	$\cos\theta$	$-\sin\theta$	0	0	0	1
$\hat{\mathbf{r}}$	$\sin\theta\cos\phi$	$\sin\theta\sin\phi$	$\cos\theta$	1	0	0	$\sin\theta$	0	$\cos\theta$
$\hat{\boldsymbol{\theta}}$	$\cos\theta\cos\phi$	$\cos\theta\sin\phi$	$-\sin\theta$	0	1	0	$\cos\theta$	0	$-\sin\theta$
$\hat{\boldsymbol{\phi}}$	$-\sin\phi$	$\cos\phi$	0	0	0	1	0	1	0
$\hat{\mathbf{R}}$	$\cos\phi$	$\sin\phi$	0	$\sin\theta$	$\cos\theta$	0	1	0	0
$\hat{\boldsymbol{\phi}}$	$-\sin\phi$	$\cos\phi$	0	0	0	1	0	1	0
$\hat{\mathbf{z}}$	0	0	1	$\cos\theta$	$-\sin\theta$	0	0	0	1

ELEMENTARY SOURCES AND FIELDS

Source Type	Vector Field $\tilde{\mathbf{V}}$	Potential Functions $\tilde{\Phi}$	Streamfunction $\tilde{\Psi}$
Uniform Plane Field	$V_0\hat{\mathbf{x}}$	$-V_0 x$	$V_0 y$
Uniform Axisymmetric Field	$V_0\hat{\mathbf{z}}$	$-V_0 z$	$\psi_p = \frac{1}{2}V_0 r^2$
Line Flux Source	$\frac{\tilde{F}'}{2\pi R}\hat{R}$	$-\frac{\tilde{F}'}{2\pi}\ell n R$	$\frac{\tilde{F}'}{2\pi}\varphi$
Point Flux Source	$\frac{\tilde{F}}{4\pi r^2}\hat{r}$	$\frac{\hat{F}}{4\pi r}$	$\psi_p = -\frac{F}{4\pi}\cos\theta$
Line Circulation Source	$\frac{\tilde{\Gamma}}{2\pi R}\hat{\varphi}$	$-\frac{\tilde{\Gamma}}{2\pi}\varphi$	$-\frac{\tilde{\Gamma}}{2\pi}\ell n R$
Plane (2D) Dipole	$\frac{d}{2\pi R^2}\left(\cos\varphi\hat{R} + \sin\varphi\hat{\varphi}\right)$	$\frac{d}{2\pi}\frac{\cos\varphi}{R}$	$\frac{d}{2\pi}\frac{\sin\varphi}{R}$
Point (3D) Dipole	$\frac{p}{4\pi r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$	$\frac{p}{4\pi}\frac{\cos\theta}{r^2}$	$\psi_p = \frac{p}{4\pi}\frac{\sin^2\theta}{r}$

∇-operations in Different Coordinate Systems

Cartesian Coordinates

Gradient : $\nabla\tilde{\Phi} = \hat{\mathbf{x}}\frac{\partial\tilde{\Phi}}{\partial x} + \hat{\mathbf{y}}\frac{\partial\tilde{\Phi}}{\partial y} + \hat{\mathbf{z}}\frac{\partial\tilde{\Phi}}{\partial z}.$

Divergence : $\nabla\cdot\tilde{\mathbf{V}} = \frac{\partial\tilde{V}_x}{\partial x} + \frac{\partial\tilde{V}_y}{\partial y} + \frac{\partial\tilde{V}_z}{\partial z}.$

Curl : $\nabla\times\tilde{\mathbf{V}} = \hat{\mathbf{x}}\left(\frac{\partial\tilde{V}_z}{\partial y} - \frac{\partial\tilde{V}_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial\tilde{V}_x}{\partial z} - \frac{\partial\tilde{V}_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial\tilde{V}_y}{\partial x} - \frac{\partial\tilde{V}_x}{\partial y}\right)$

Laplacian : $\nabla^2\tilde{\Phi} = \frac{\partial^2\tilde{\Phi}}{\partial^2 x} + \frac{\partial^2\tilde{\Phi}}{\partial^2 y} + \frac{\partial^2\tilde{\Phi}}{\partial^2 z}.$

Cylindrical Coordinates

Gradient : $\nabla\tilde{\Phi} = \hat{\mathbf{R}}\frac{\partial\tilde{\Phi}}{\partial R} + \hat{\boldsymbol{\varphi}}\frac{1}{R}\frac{\partial\tilde{\Phi}}{\partial\varphi} + \hat{\mathbf{z}}\frac{\partial\tilde{\Phi}}{\partial z}.$

Divergence : $\nabla\cdot\tilde{\mathbf{V}} = \frac{1}{R}\frac{\partial}{\partial R}(R\tilde{V}_R) + \frac{1}{R}\frac{\partial}{\partial\varphi}\tilde{V}_\varphi + \frac{\partial}{\partial z}\tilde{V}_z.$

Curl : $\nabla\times\tilde{\mathbf{V}} = \hat{\mathbf{R}}\left[\frac{1}{R}\frac{\partial\tilde{V}_z}{\partial\varphi} - \frac{\partial\tilde{V}_\varphi}{\partial z}\right] + \hat{\boldsymbol{\varphi}}\left[\frac{\partial\tilde{V}_R}{\partial z} - \frac{\partial\tilde{V}_z}{\partial R}\right] + \hat{\mathbf{z}}\left[\frac{1}{R}\frac{\partial}{\partial R}(R\tilde{V}_\varphi) - \frac{1}{R}\frac{\partial\tilde{V}_R}{\partial\varphi}\right]$

Laplacian : $\nabla^2\tilde{\Phi} = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\tilde{\Phi}}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2\tilde{\Phi}}{\partial\varphi^2} + \frac{\partial^2\tilde{\Phi}}{\partial z^2}.$

Spherical Coordinates

Gradient : $\nabla\tilde{\Phi} = \hat{\mathbf{r}}\frac{\partial\tilde{\Phi}}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial\tilde{\Phi}}{\partial\theta} + \hat{\boldsymbol{\varphi}}\frac{1}{r\sin\theta}\frac{\partial\tilde{\Phi}}{\partial\varphi}.$

Divergence : $\nabla\cdot\tilde{\mathbf{V}} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tilde{V}_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\tilde{V}_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\tilde{V}_\varphi.$

Curl : $\nabla\times\tilde{\mathbf{V}} = \hat{\mathbf{r}}\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\tilde{V}_\varphi) - \frac{\partial\tilde{V}_\theta}{\partial\varphi}\right] + \hat{\boldsymbol{\theta}}\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial\tilde{V}_r}{\partial\varphi} - \frac{\partial}{\partial r}(r\tilde{V}_\varphi)\right] + \hat{\boldsymbol{\varphi}}\frac{1}{r}\left[\frac{\partial}{\partial r}(r\tilde{V}_\theta) - \frac{\partial\tilde{V}_r}{\partial\theta}\right]$

Laplacian : $\nabla^2\tilde{\Phi} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\tilde{\Phi}}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\tilde{\Phi}}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\tilde{\Phi}}{\partial\varphi^2}.$

Line, Surface and Volume Elements in Different Coordinate Systems

Cartesian Coordinates

$$
\mathrm{d}\boldsymbol{\ell} = \hat{\mathbf{x}}\mathrm{d}x + \hat{\mathbf{y}}\mathrm{d}y + \hat{\mathbf{z}}\mathrm{d}z
$$

$$
\mathrm{d}\mathbf{S} = \begin{cases} \hat{\mathbf{x}}\,\mathrm{d}y\mathrm{d}z, & \text{yz-plane} \\ \hat{\mathbf{y}}\,\mathrm{d}x\mathrm{d}z, & \text{xz-plane} \\ \hat{\mathbf{z}}\,\mathrm{d}x\mathrm{d}y, & \text{xy-plane} \end{cases}
$$

$$
\mathrm{d}V = \mathrm{d}x\mathrm{d}y\mathrm{d}z
$$

Cylindrical Coordinates

$$
\mathrm{d}\boldsymbol{\ell} = \hat{\mathbf{R}}\mathrm{d}R + \hat{\boldsymbol{\varphi}}\,R\,\mathrm{d}\varphi + \hat{\mathbf{z}}\mathrm{d}z
$$

$$
\mathrm{d}\mathbf{S} = \begin{cases} \hat{\mathbf{R}}\,R\,\mathrm{d}\varphi\mathrm{d}z, & \text{curved-surface} \\ \hat{\boldsymbol{\varphi}}\,\mathrm{d}R\mathrm{d}z, & \text{meridional-plane} \\ \hat{\mathbf{z}}\,R\,\mathrm{d}R\mathrm{d}\varphi, & \text{top/bottom-plane} \end{cases}
$$

$$
\mathrm{d}V = R\,\mathrm{d}R\mathrm{d}\varphi\mathrm{d}z
$$

Spherical Coordinates

$$
\mathrm{d}\boldsymbol{\ell} = \hat{\mathbf{r}}\mathrm{d}r + \hat{\boldsymbol{\theta}}\,r\sin\theta\mathrm{d}\theta + \hat{\boldsymbol{\varphi}}\,r\mathrm{d}\varphi
$$

$$
\mathrm{d}\mathbf{S} = \begin{cases} \hat{\mathbf{r}}\,r^2\sin\theta\mathrm{d}\theta\mathrm{d}\varphi, & \text{curved-surface} \\ \hat{\boldsymbol{\theta}}\,r\sin\theta\mathrm{d}r\mathrm{d}\varphi, & \text{meridional-plane} \\ \hat{\boldsymbol{\varphi}}\,r\mathrm{d}r\mathrm{d}\theta, & \text{xy-plane} \end{cases}
$$

$$
\mathrm{d}V = r^2\sin\theta\,\mathrm{d}r\mathrm{d}\theta\mathrm{d}\varphi
$$