

ENGS 200 / PHYS 110. Methods in Applied Mathematics II

Offered: 10W

Continuation of ENGS/PHYS 100 with emphasis on variational calculus, integral equations, and asymptotic and perturbation methods for integrals and differential equations. Selected topics include functional differentiation, Hamilton's principles, Rayleigh-Ritz method, Fredholm and Volterra equations, integral transforms, Schmidt-Hilbert theory, asymptotic series, methods of steepest descent and stationary phase, boundary layer theory, WKB methods, and multiple-scale theory.

Prerequisite: ENGS/PHYS 100 or equivalent

Instructor: William Lotko

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Class time: MWF 1:45 – 2:50p, X-Hour Thu 1 - 1:50p

Classroom: 200 Cummings

Evaluation criteria: Weekly Exercises (50%); Final Exam (50%)

Textbooks:

- [*Advanced Mathematical Methods for Scientists and Engineers I: Asymptotic Methods and Perturbation Theory*](#) by Carl M. Bender, Steven A. Orszag, Springer (1999). Chapters 6-11 of this highly regarded textbook contain reference material, including many applications of the theory, for the lectures on Asymptotic Analysis of Integrals and Global Perturbation Methods.
- [*Mathematical Methods for Physics and Engineering, 3rd Edition*](#) by K.F. Riley, M.P. Hobson and S.J. Bence, Cambridge University Press (2006). This textbook has been used for the Fall 2008, 2009 offerings of ENGS/PHYS 100. Lectures on the Calculus of Variations and Integral Equations will draw on material from Chapters 22 and 23 of this book. An excellent, alternative reference for this material is contained in Chapters 2 and 3 of [*Methods of Applied Mathematics, 2nd Edition*](#) by Francis B. Hildebrand, Dover (1992)

Lecture Topics

Week 1-2	Asymptotic Analysis	
Monday Jan 4	Order; asymptotic expansion; convergent vs. divergent series	BO 6.1-6.3
Wednesday Jan 6	Watson's Lemma	BO 6.4
Friday Jan 8	Laplace's method	BO 6.4
Monday Jan 11	Method of stationary phase	BO 6.5
Wednesday Jan 13	Method of steepest descents	BO 6.6
Friday Jan 15	Application of steepest decent method	BO 6.6

Week 3-5	Global Perturbation Methods	
Monday Jan 18	No class - Martin Luther King, Jr. Day	
Wednesday Jan 20	Regular/singular perturbation theory	BO 7
Friday Jan 22	Examples where a regular perturbation expansion fails	BO 7
Monday Jan 25	Matched asymptotic expansions; boundary layer analysis	BO 9.1-9.5
Wednesday Jan 27	Internal boundary layer	BO 9.6
Friday Jan 29	Nonlinear boundary layer	BO 9.7
Monday Feb 1	WKB theory	BO 10.1-10.3
Wednesday Feb 3	Turning points; tunneling	BO 10.4-10.6
Friday Feb 5	Multiple-scale method	BO 11
Monday Feb 8	Poincaré-Lindstedt method	Notes
Week 6-8	Calculus of Variations	
Wednesday Feb 10	Functionals; necessary conditions for extrema; variational derivative	Notes H 2.1-2.3
Monday Feb 15	Euler-Lagrange equation; first integrals; examples	Notes, H 2.11 RHB 22.1,22.2
Wednesday Feb 17	Higher derivatives; several functions; multiple integrals	H 2.6 RHB22.3
Friday Feb 19	Natural boundary conditions; transition conditions; variable endpoints	H 2.4, 2.7 RHB 22.3
Monday Feb 22	Constraints; Lagrange multipliers	H 2.7 RHB 22.4
Wednesday Feb 24	Hamilton's Principle; canonical formulation	H 2.10 RHB 22.5
Friday Feb 26	Rayleigh-Ritz method; eigenvalue problem	H 2.9, 2.19 RHB 22.6-22.8

Week 9-10	Integral Equations	
Monday Mar 1	Classification; origin; relation to differential equations	H 3.1-3.5 RHB23.1-23.3
Wednesday Mar 3	Fredholm equations; Hilbert-Schmidt method	H 3.6-3.8 RHB 23.4, 23.7
Friday Mar 5	Volterra equations; Iterative solution	H 3.9-3.10 RHB 23.5,23.6
Monday Mar 8	Transform methods	Notes H 3.12-3.13 RHB 23.4

Extended Reference List

General Mathematical Methods including Course Topics

- Arfken, G.B. and H.J. Weber, *Mathematical Methods for Physicists*, 6th Edition, Elsevier Academic Press (2005)
 Courant, R. and D. Hilbert, *Methods of Mathematical Physics*, Wiley-Interscience, New York (1953)
 Greenberg, M.D., *Foundations of Applied Mathematics*, Prentice-Hall Inc., Englewood Cliffs, N.J. (1978)
 Logan, J.D., *Applied Mathematics*, 3rd Edition, Wiley-Interscience (2006)
 Riley, K.F., M.P. Hobson and S.J. Bence, *Mathematical Methods for Physics and Engineering*, 3rd ed., Cambridge University Press (2006)

Asymptotic Methods

- Awrejcewicz, J. and V.A. Krysko, *Introduction to Asymptotic Methods*, Chapman & Hall/CRC (2006)
 Bender, C.M. and S.A. Orszag, *Advanced Mathematical Methods for Scientists and Engineers I*, Springer (1999).
 Bush, A.W. *Perturbation Methods for Engineers and Scientists*, CRC Press, Boca Raton (1992)
 Hinch, E.J., *Perturbation Methods*, Cambridge University Press, New York (1991)
 Holmes, M.H., *Introduction to Perturbation Methods*, Springer (1995)
 Kevorkian, J. and J.D. Cole, *Perturbation Methods in Applied Mathematics*, Springer (1985)
 Miller, P.D., *Applied Asymptotic Analysis*, American Mathematical Society V. 75 (2006)
 Murdock, J.A., *Perturbations: Theory and Methods*, SIAM (1999)
 Murray, J.D., *Asymptotic Analysis*, Springer-Verlag Inc., New York (1984)
 Nayfeh, A.H., *Introduction to Perturbation Methods*, Wiley Interscience, New York (1981)
 Kevorkian, J. and J.D. Cole, *Perturbation in Applied Mathematics*, Springer-Verlag, New York (1981)
 VanDyke, M., *Perturbation Methods in Fluid Mechanics*, Parabolic Press, Stanford, CA (1975)

Calculus of Variations

- Brechtken-Manderschild, U., *Introduction to the Calculus of Variations*, Chapman and Hall, New York, (1991)
 Hildebrand, F.B., *Methods of Applied Mathematics*, 2nd Edition, Dover (1992)
 Troutman, J.L., *Variational Calculus and Optimal Control*, Springer (1996)
 Weinstock, R., *Calculus of Variations*, Dover Publications, New York (1974)

Integral Equations

- Hochstadt, H. *Integral Equations*, John Wiley and Sons, Inc., New York (1983)
 Jerri, A.J., *Introduction to Integral Equations with Applications*, 2nd ed., Wiley-Interscience (1999)
 Pipkin, A.C., *A Course on Integral Equations*, Springer-Verlag, (1991)
 Porter, D., and D.S.G. Sterling, *Integral Equations*, Cambridge University Press (1990)
 Tricomi, F.G., *Integral Equations*, Dover (1983)
 Wazwaz, A.-M., *A First Course in Integral Equations*, World Scientific (1997)

Handbooks

- Abramowitz, M. and I.A. Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, Dover (1972)
 Erdelyi, A., W. Magnus, F. Oberhettinger, and F.G. Tricomi, *Tables of Integral Transforms, Vol I and II*, McGraw Hill (195)
 Gradshteyn, I.S., and Ryzhik, I.W., *Tables of Integrals, Series, and Products* (5th Edition), Academic Press, Inc. (1994)